Spatial Inference –
Learning vs. Constraint Solving

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Spatial Relations

Constraint Solving

Machine Learning

Introduction.

- Starting point: textual descriptions of spatial scenes.

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- Aims:

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- Aims:
  - Checking the texts for consistency.

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  - Checking the texts for consistency.
  - Generating appropriate depictions.

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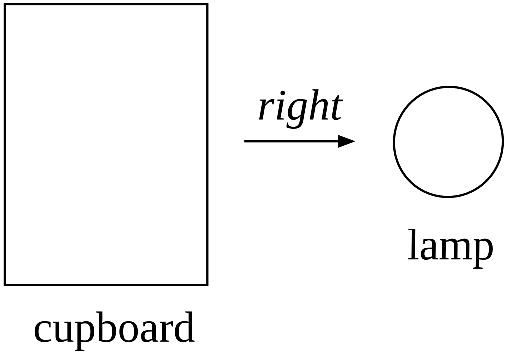
- Aims:
  - Checking the texts for consistency.
  - Generating appropriate depictions.
  - Inferring new information (not initially given in the texts).
- Here: consider only depiction generation.

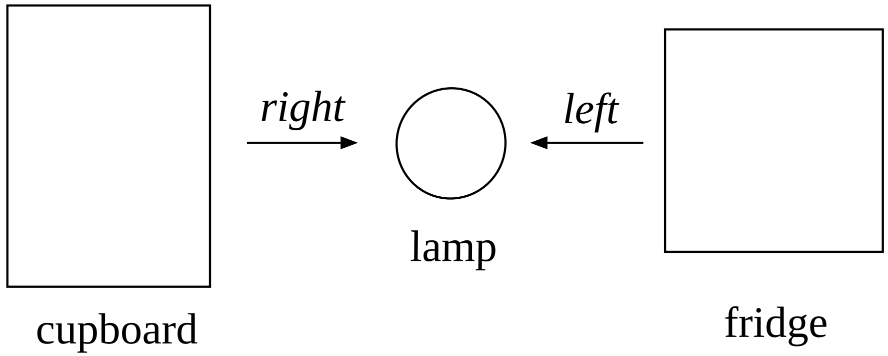
Introduction: Spatial Relations.

(1) (2) (3)



`right(cupboard,lamp)`

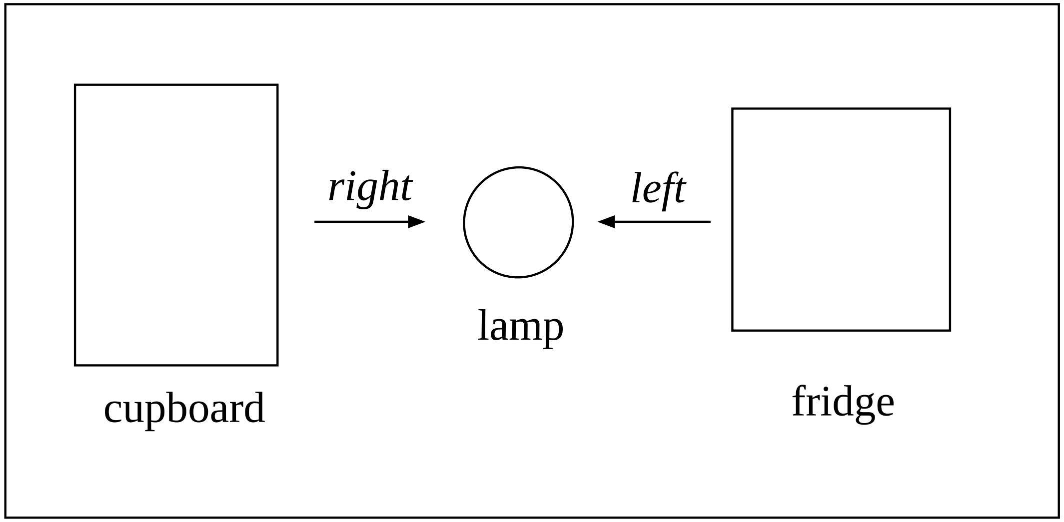
Introduction: Spatial Relations.




```
right(cupboard,lamp)
left(fridge,lamp)
```

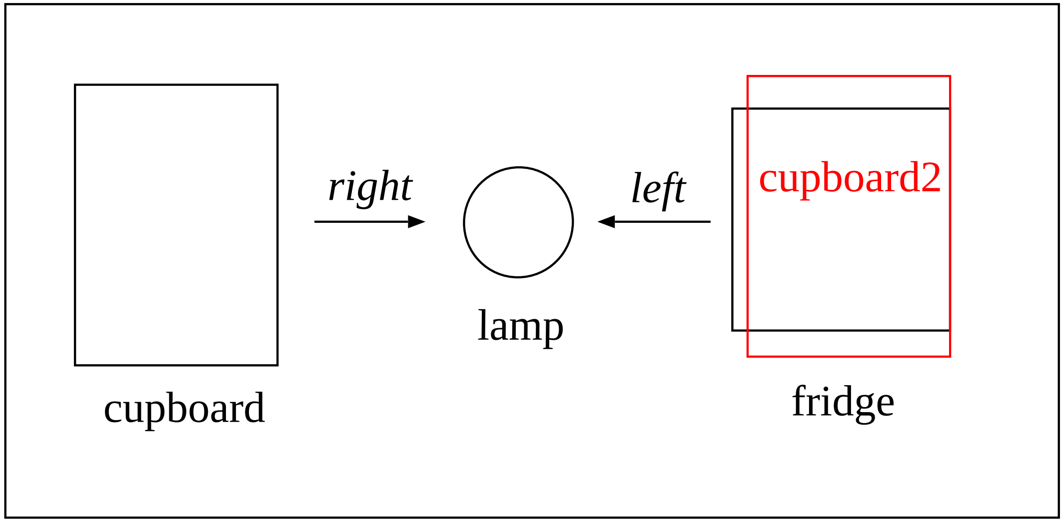
Introduction: Spatial Relations.

(1) (2) (3)



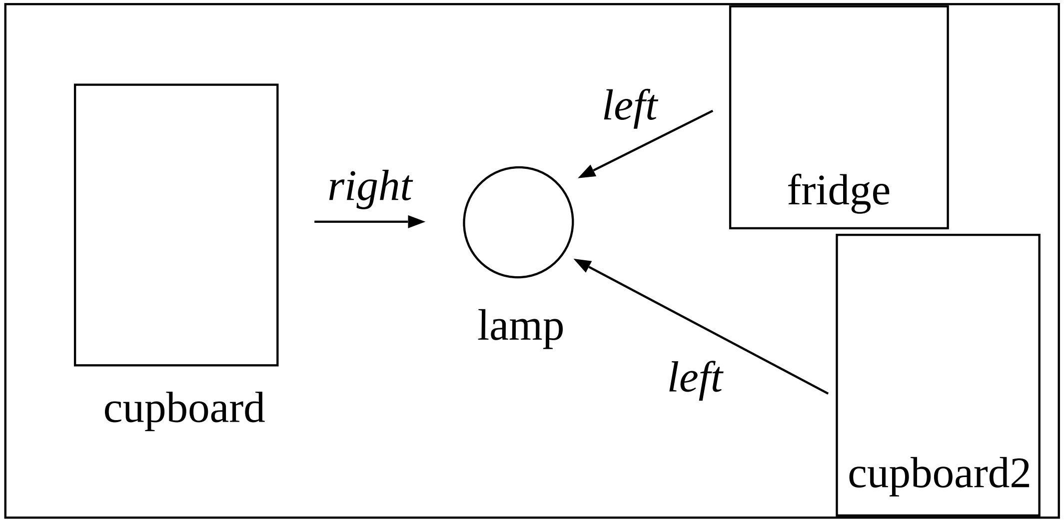

```
right(cupboard,lamp)
left(fridge,lamp)
in_room(fridge),in_room(lamp),in_room(cupboard)
```

Introduction: Spatial Relations.




```
right(cupboard,lamp)
left(fridge,lamp)
in_room(fridge),in_room(lamp),in_room(cupboard)
left(cupboard2,lamp),in_room(cupboard2)
```

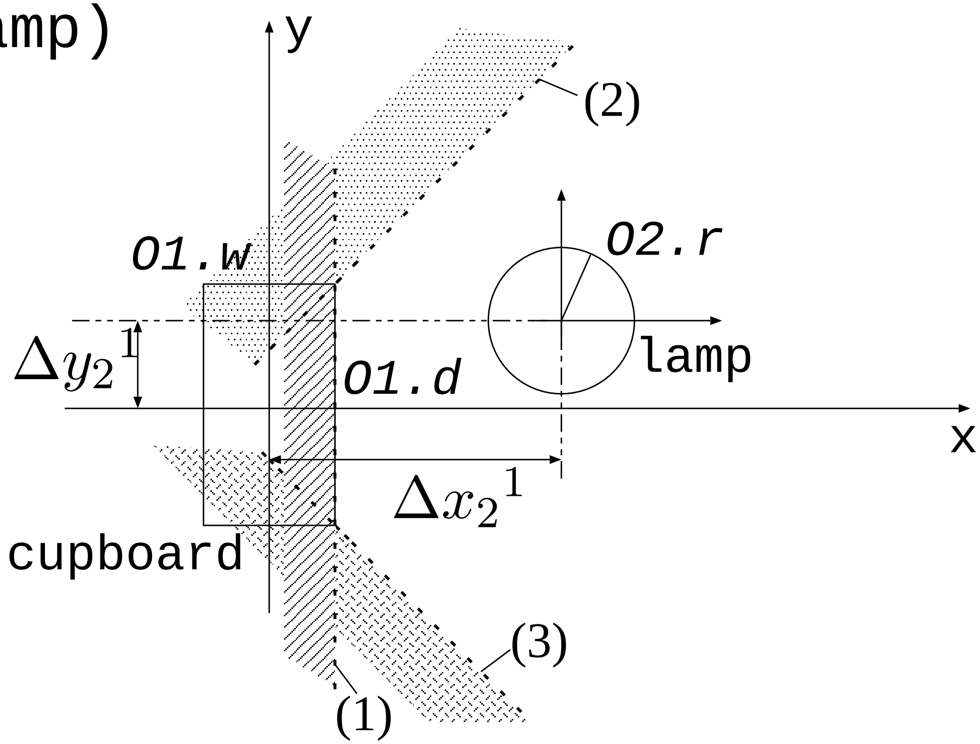
Introduction: Spatial Relations.




```
right(cupboard,lamp)
left(fridge,lamp)
in_room(fridge),in_room(lamp),in_room(cupboard)
left(cupboard2,lamp),in_room(cupboard2)
```

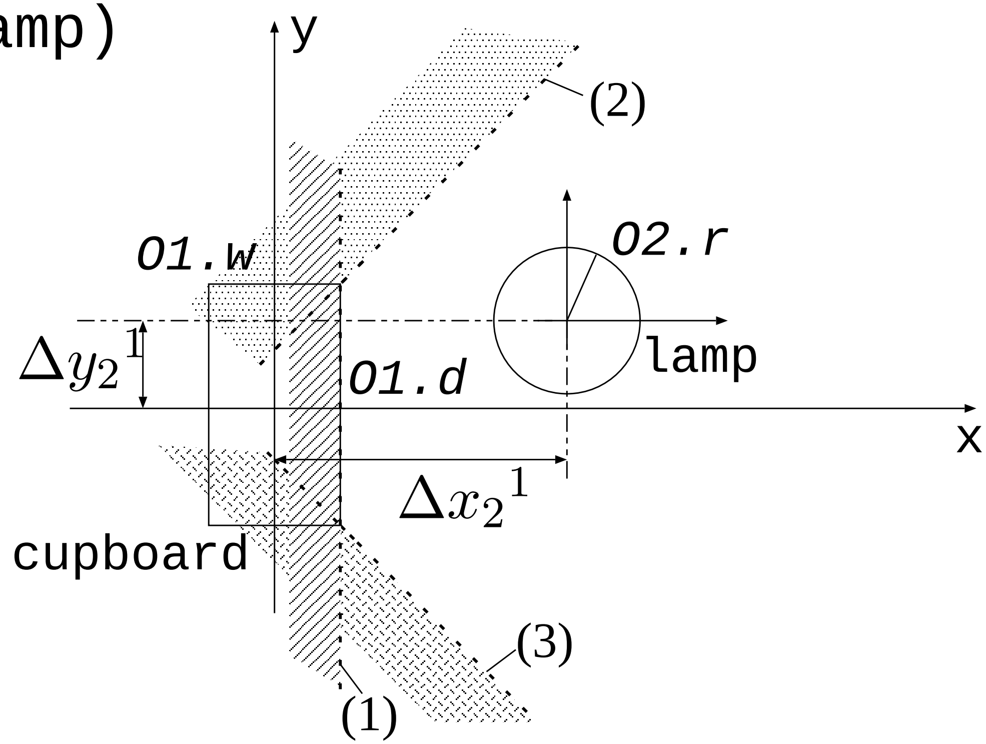
Introduction: Spatial Relations. A Metric Approach. **(1) (2) (3)**

```
right(cupboard, lamp)
```



Introduction: Spatial Relations. A Metric Approach. **(1) (2) (3)**

`right(cupboard, lamp)`



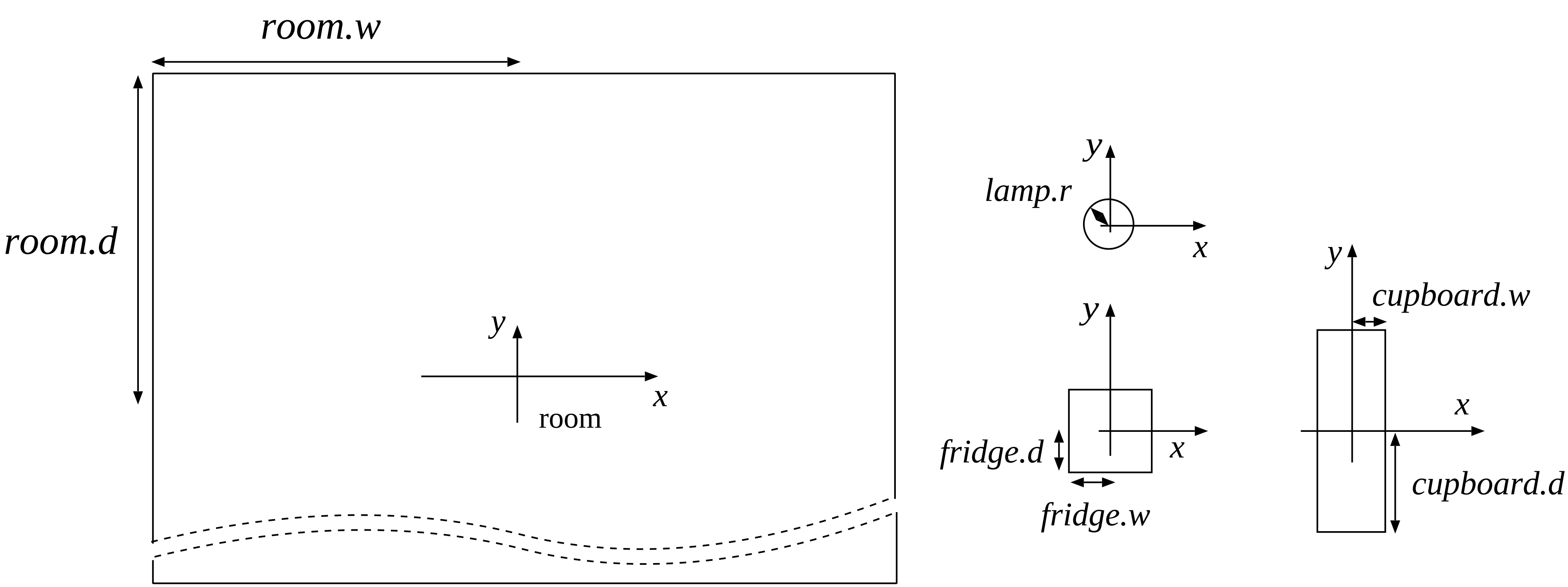
Mathematical description: `right(`$O1$`, `$O2$`)`

$$\Delta x_2{}^1 \geq O1.w + O2.r \qquad (1)$$

$$\Delta x_2{}^1 \geq \Delta y_2{}^1 + O1.w - O1.d + \sqrt{2}O2.r \qquad (2)$$

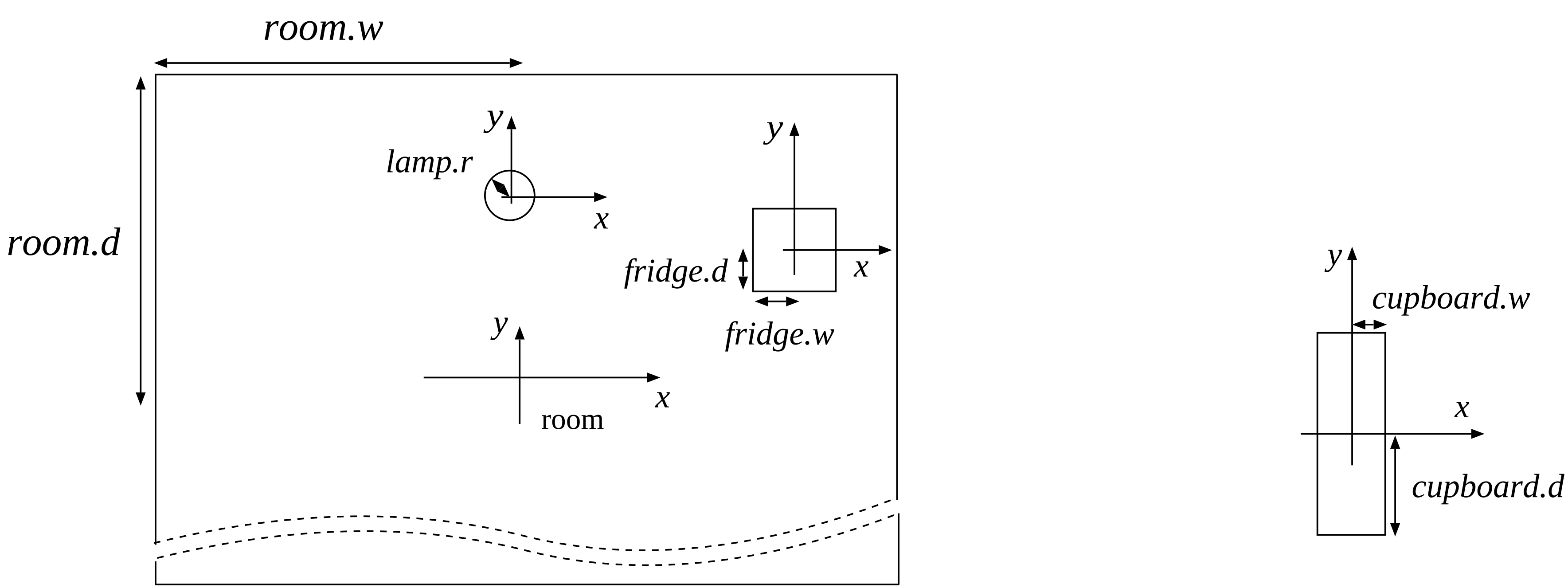
$$\Delta x_2{}^1 \geq -\Delta y_2{}^1 + O1.w - O1.d + \sqrt{2}O2.r \qquad (3)$$

Introduction: Spatial Relations. Example Scene. **(1) (2) (3)**




```
room.w∈[4.0,4.5],room.d∈[4.0,4.5],lamp.r=0.3,
fridge.w∈[0.4,0.5],fridge.d∈[0.4,0.5],
cupboard.w=0.4, cupboard.d∈[1.0,1.2]
in_room(fridge),...,not_overlap(fridge,lamp,...),
```

Introduction: Spatial Relations. Example Scene.



room.w∈[4.0,4.5],room.d∈[4.0,4.5],lamp.r=0.3,
fridge.w∈[0.4,0.5],fridge.d∈[0.4,0.5],
cupboard.w=0.4, cupboard.d∈[1.0,1.2]
in_room(fridge),...,not_overlap(fridge,lamp,...),
left(fridge,lamp),

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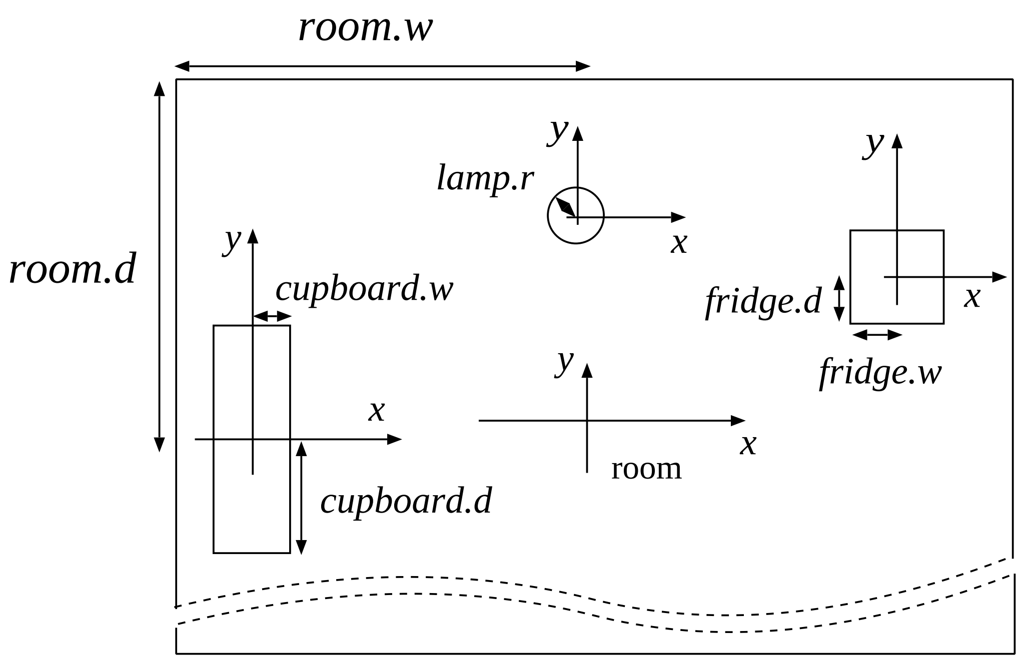

room.w∈[4.0,4.5],room.d∈[4.0,4.5],lamp.r=0.3,
fridge.w∈[0.4,0.5],fridge.d∈[0.4,0.5],
cupboard.w=0.4, cupboard.d∈[1.0,1.2]
in_room(fridge),...,not_overlap(fridge,lamp,...),
left(fridge,lamp),right (cupboard, lamp)

Outline

- Constraint Solving:

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  - Express spatial relations mathematically by constraints.

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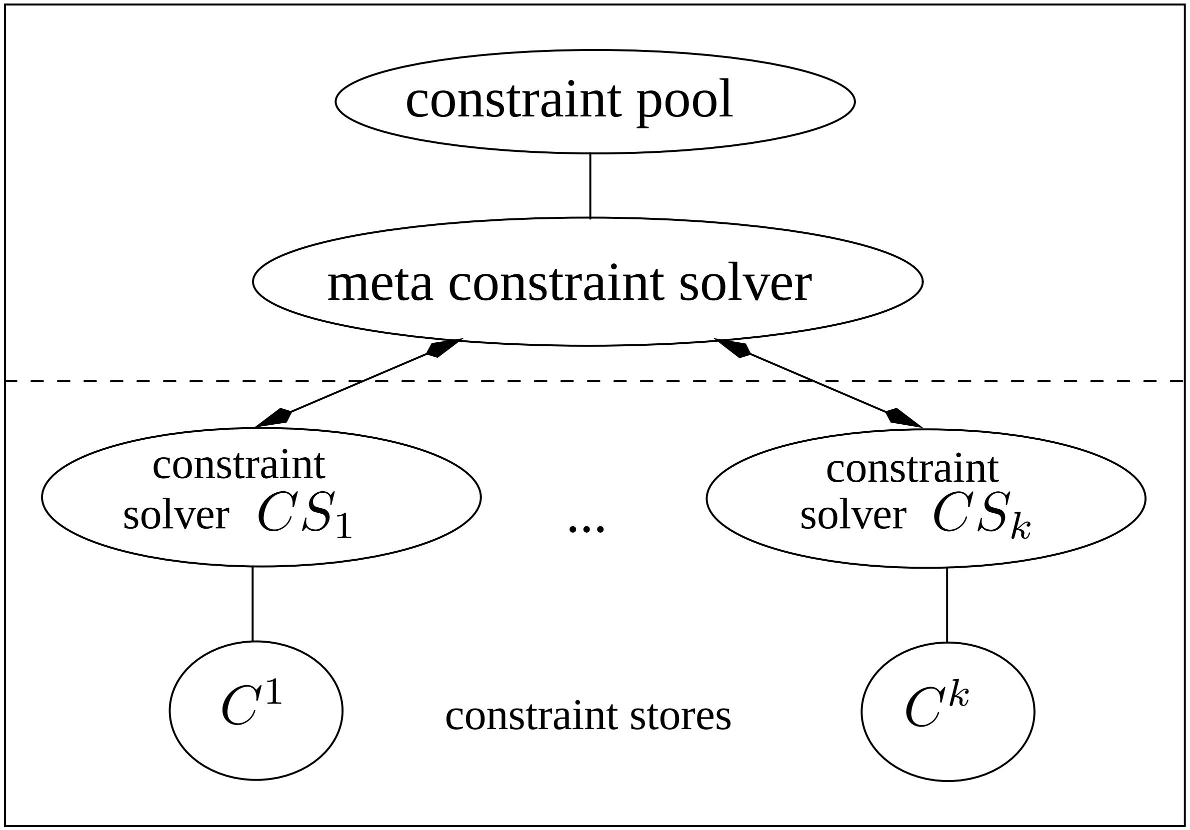
- Constraint Solving:
  - Express spatial relations mathematically by constraints.
  - Determine constraint domains.
  - Solve constraints by cooperating constraint solvers.
- Machine Learning:
  - Learning spatial relations.
  - Deciding constraints by depiction generation.

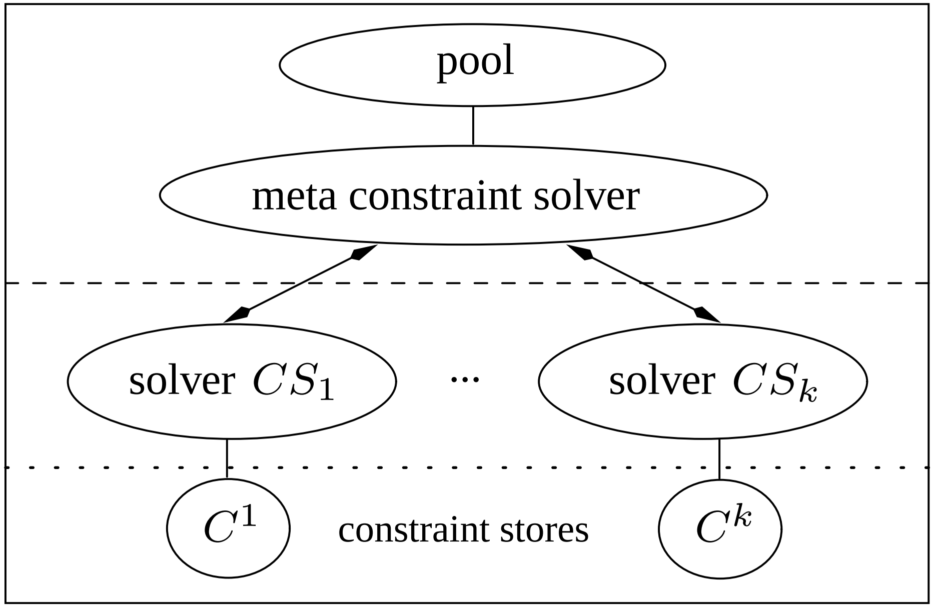
Spatial Inference: Constraint Solving.

Cooperative Constraint Solvers



Spatial Inference: Constraint Solving.

Cooperative Constraint Solvers



- Implementation: [HGS01],
- Integration of solvers, e.g. an interval arithmetic solver [Bra02], a solver for linear arithmetic [Krz97], a finite domain constraint solver [CSP01].

Constraint Solving. Constraint-File.


```
[global]
  verbose = 1
  pool    = meta.pool.Simple
  output  = all
  outvars = {roomw, roomd, dx10, dy10, dx20, dy20, dx30, dy30}
[solver]
  ISolver = solver.brandeis.Brandeis
```

Constraint Solving. Constraint-File. (1) (2) (3)

```
[global]
  verbose = 1
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  output  = all
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[solver]
  ISolver = solver.brandeis.Brandeis

[constraints]
  room.w, room.d in [4, 4.5];      # width and depth of the room
  O1.w = 0.4; O1.d in [1.0, 1.2]; # size of the cupboard
  O3.w, O3.d in [0.4, 0.5];       # size of the fridge
  O2.r = 0.3;                     # radius of the lamp
```

Constraint Solving. Constraint-File.


```
[global]
  verbose = 1
  pool    = meta.pool.Simple
  output  = all
  outvars = {roomw, roomd, dx10, dy10, dx20, dy20, dx30, dy30}
[solver]
  ISolver = solver.brandeis.Brandeis

[constraints]
  ...  # sizes of objects
  # the relation right(cupboard, lamp)
  dx21 >= O1.w + O2.r;
  dx21 >= dy21 + O1.w - O1.d + (2 ^ 0.5) * O2.r;
  dx21 >= - dy21 + O1.w - O1.d + (2 ^ 0.5) * O2.r;
```

Constraint Solving. Constraint-File. (1) (2) (3)

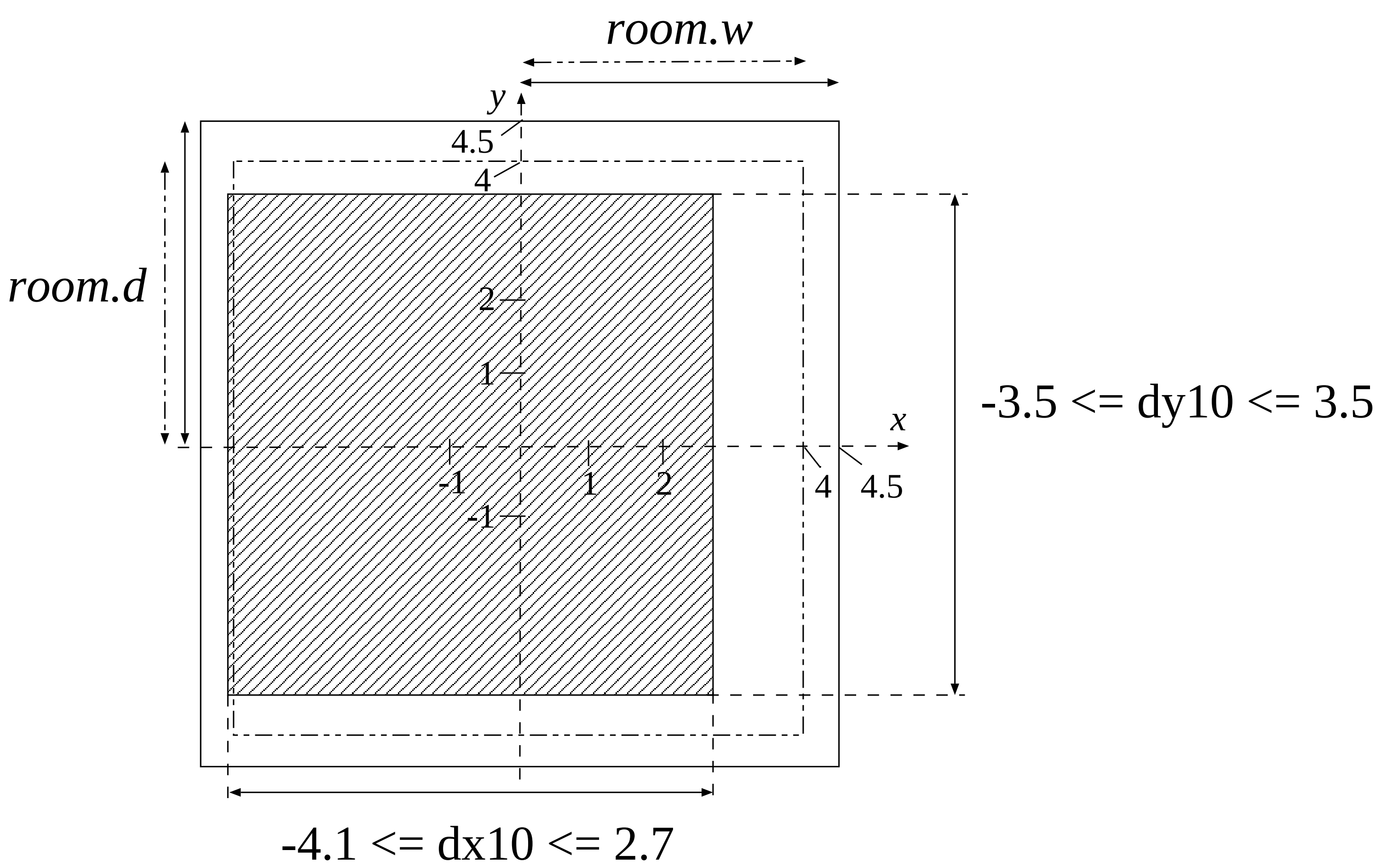
```
[global]
  verbose = 1
  pool    = meta.pool.Simple
  output  = all
  outvars = {roomw, roomd, dx10, dy10, dx20, dy20, dx30, dy30}
[solver]
  ISolver = solver.brandeis.Brandeis

[constraints]
  ...  # sizes of objects
  ...  # the relation right(cupboard, lamp)

  # the coordinates of the lamp wrt.  the cupboard (transformation)
  dx21 = dx20 - dx10;
  dy21 = dy20 - dy10;
```

Constraint Solving. Solution space.

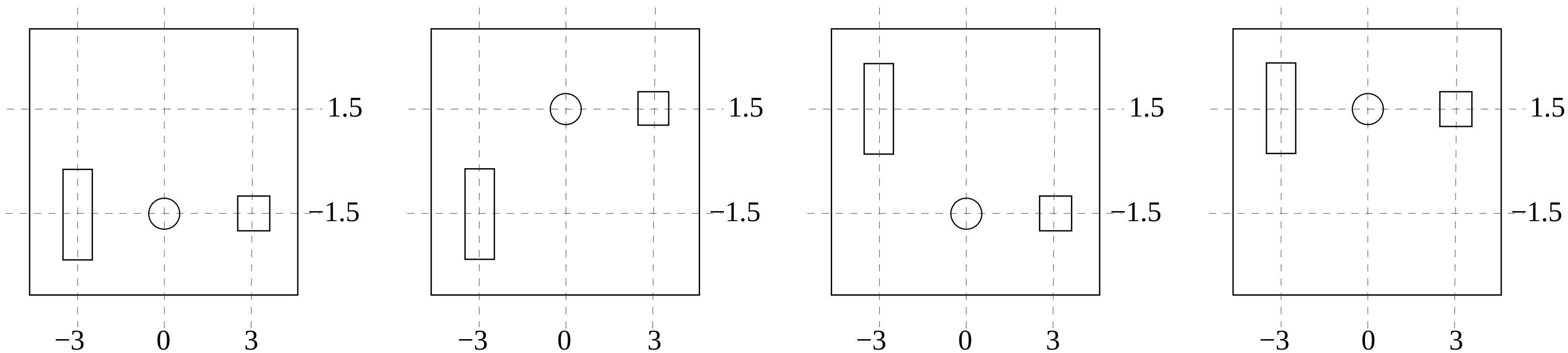
Placement of the cupboard. Solution space:



Constraint Solving. Solution space.

Solution space. Depictions:

```
...
[solver]
  ISolver = solver.brandeis.Brandeis
  FDSolver= solver.csplib.CSPlib
[constraints]
  ...
  dx10, dx20, dx30 inn {-3.0, 0.0, 3.0};
  dy10, dy20, dy30 inn {-1.5, 1.5};
```



Machine Learning Approach

- Constraint solvers are often incomplete.

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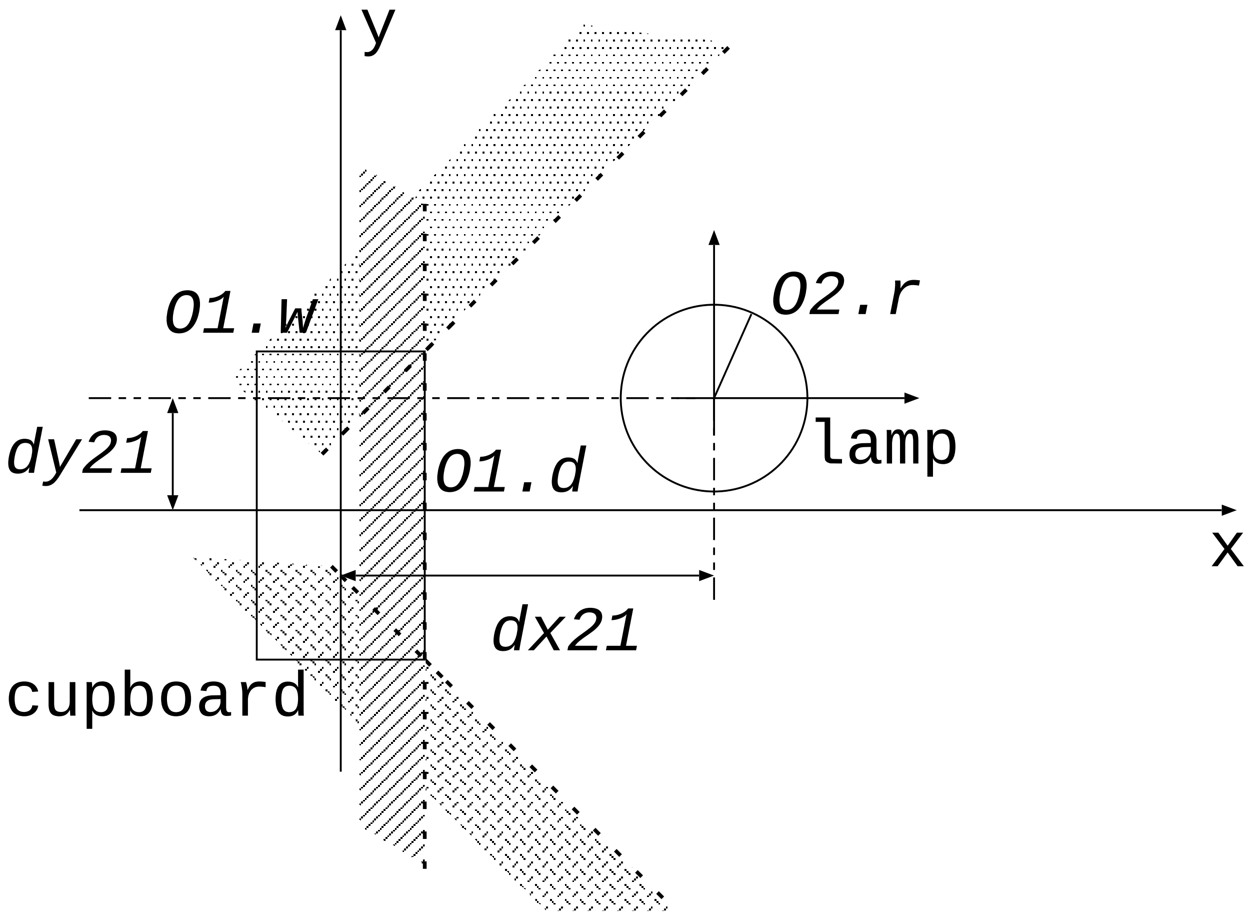
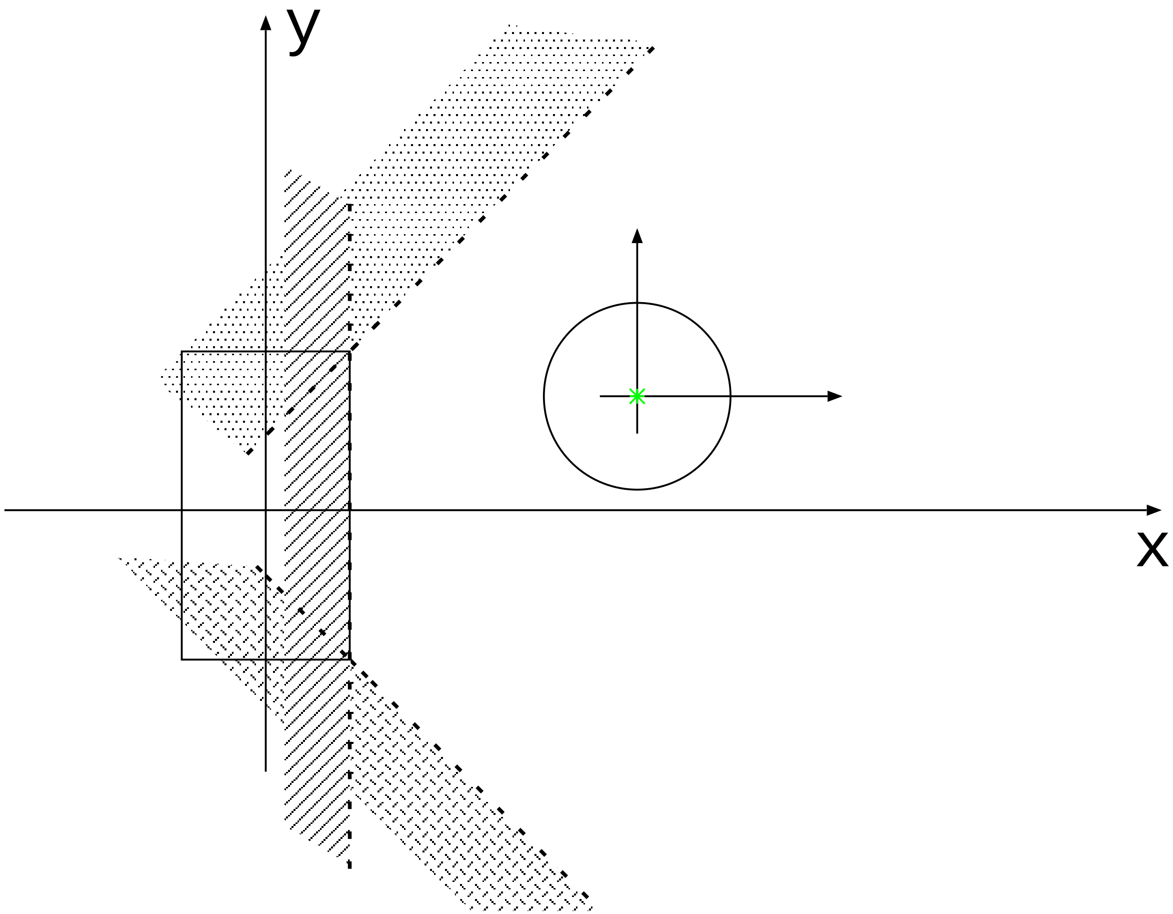
- Constraint solvers are often incomplete.
- 2-Step algorithm:
  - Learning the problem, i.e. the decision boundary.
  - Use results for solving the constraints,
    i.e. generating depictions.
- Learning step is done only once.

Machine Learning. Learning.

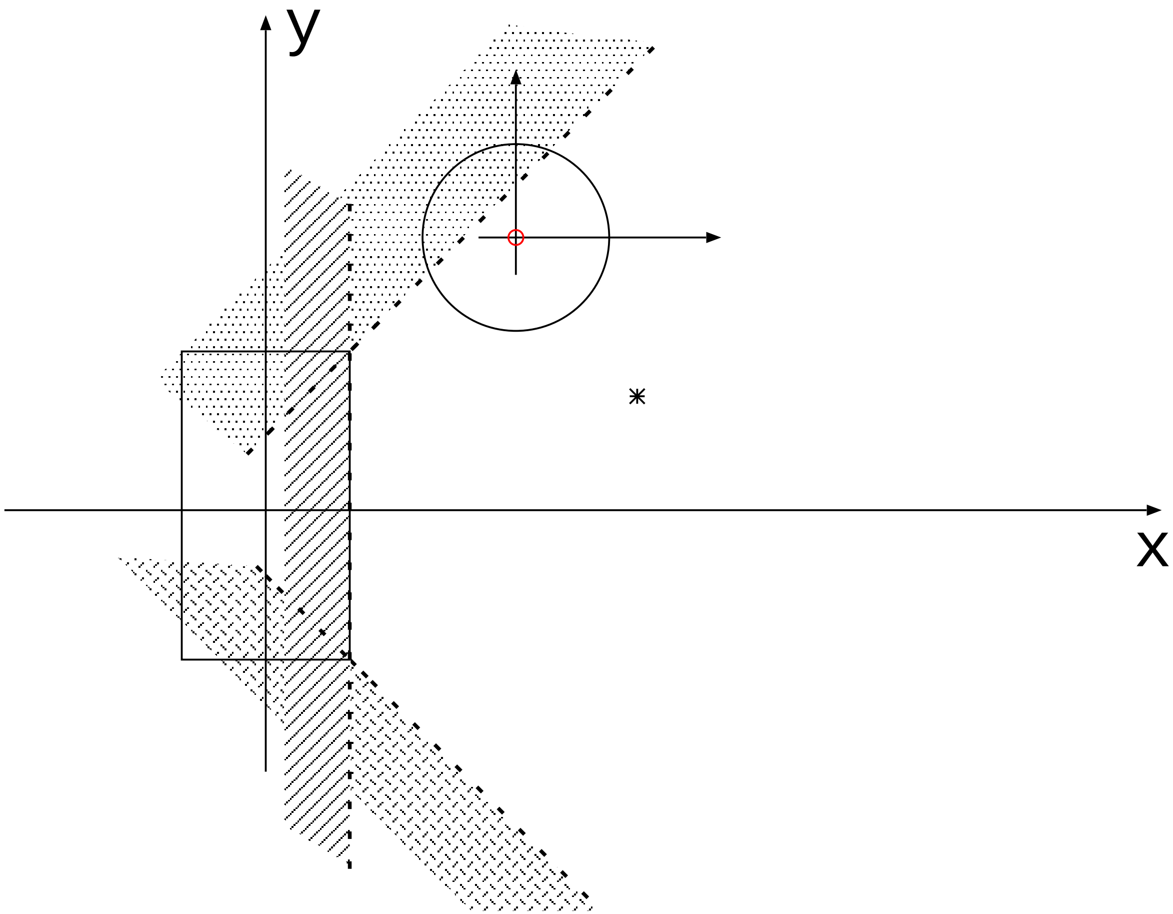


right(cupboard, lamp).

Machine Learning. Learning.

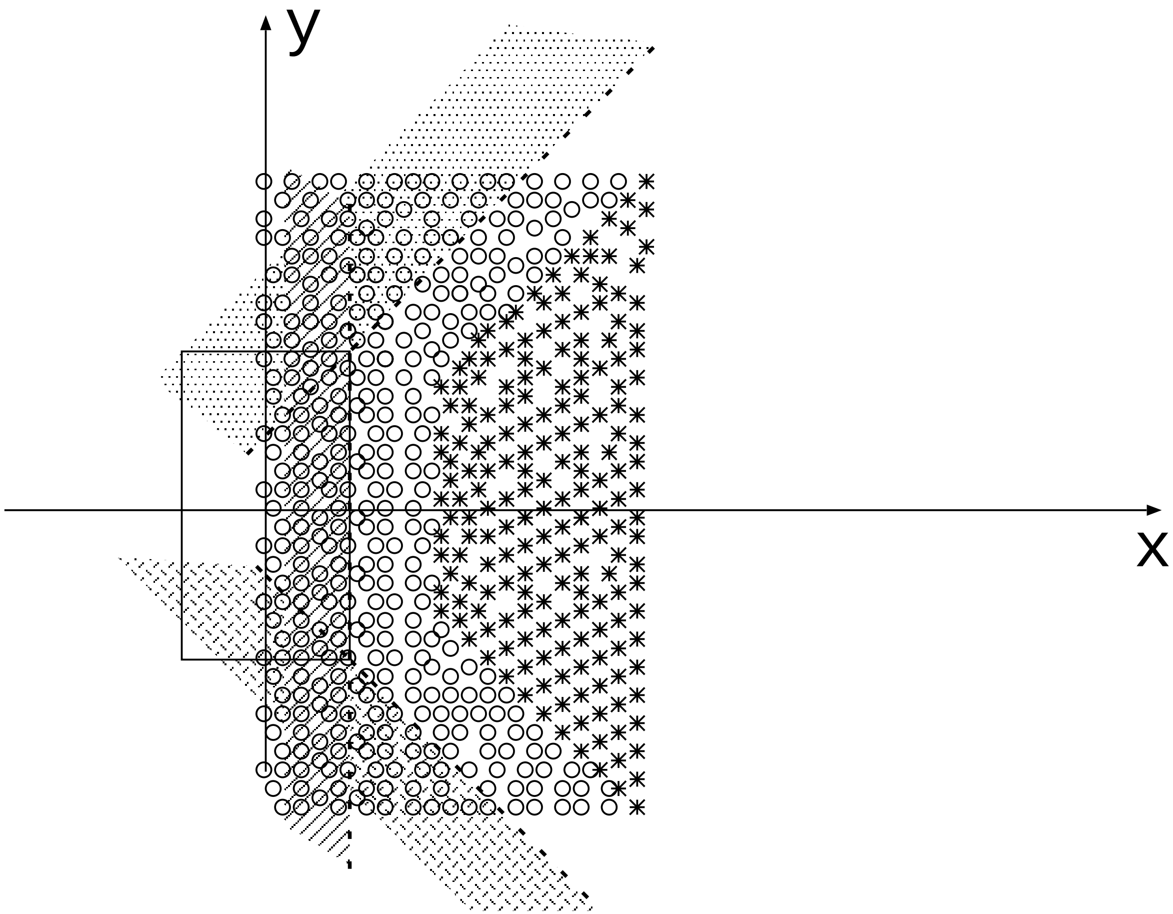


Slice of configuration space: parameters w, d, r clamped.



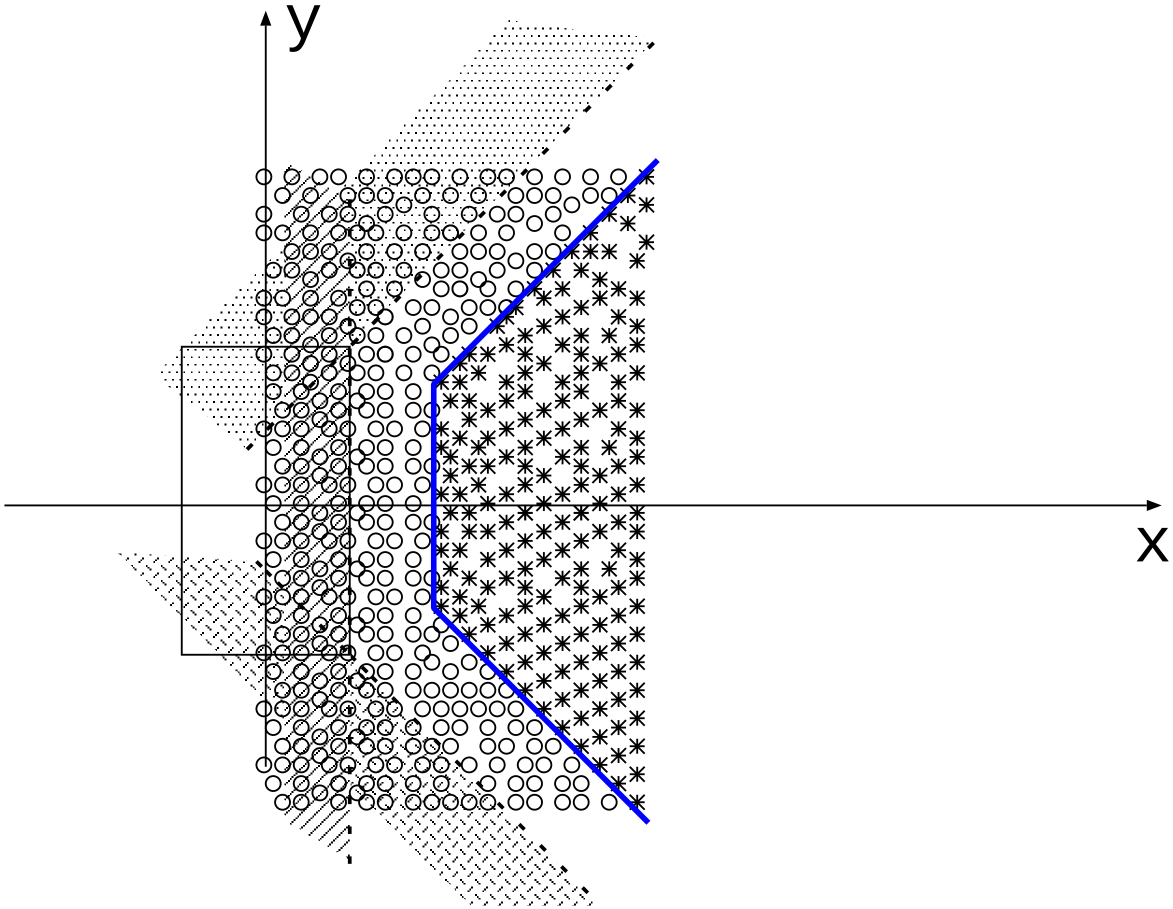
Slice of configuration space: parameters w, d, r clamped.

Machine Learning. Learning.



Dataset for training with CAL5.

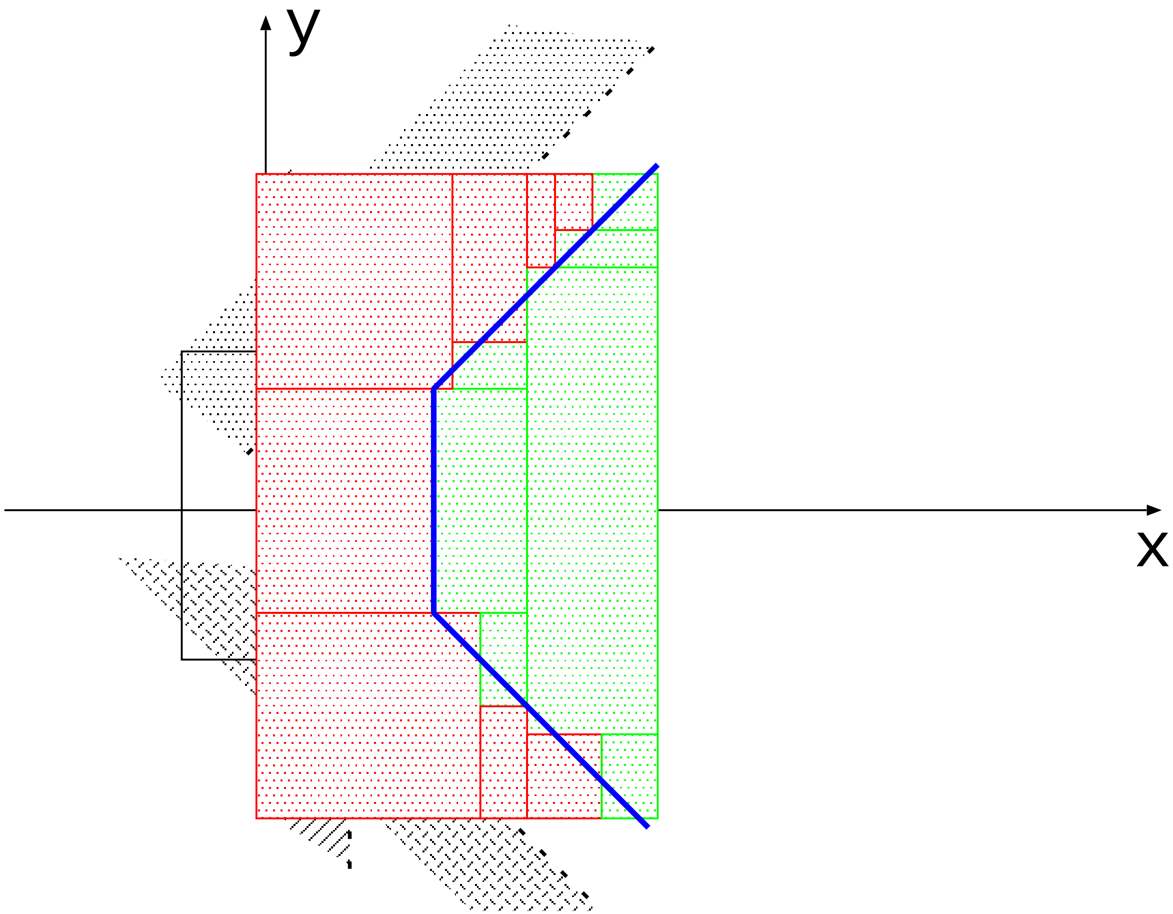
Machine Learning. Learning.



Decision boundary (to be learned).

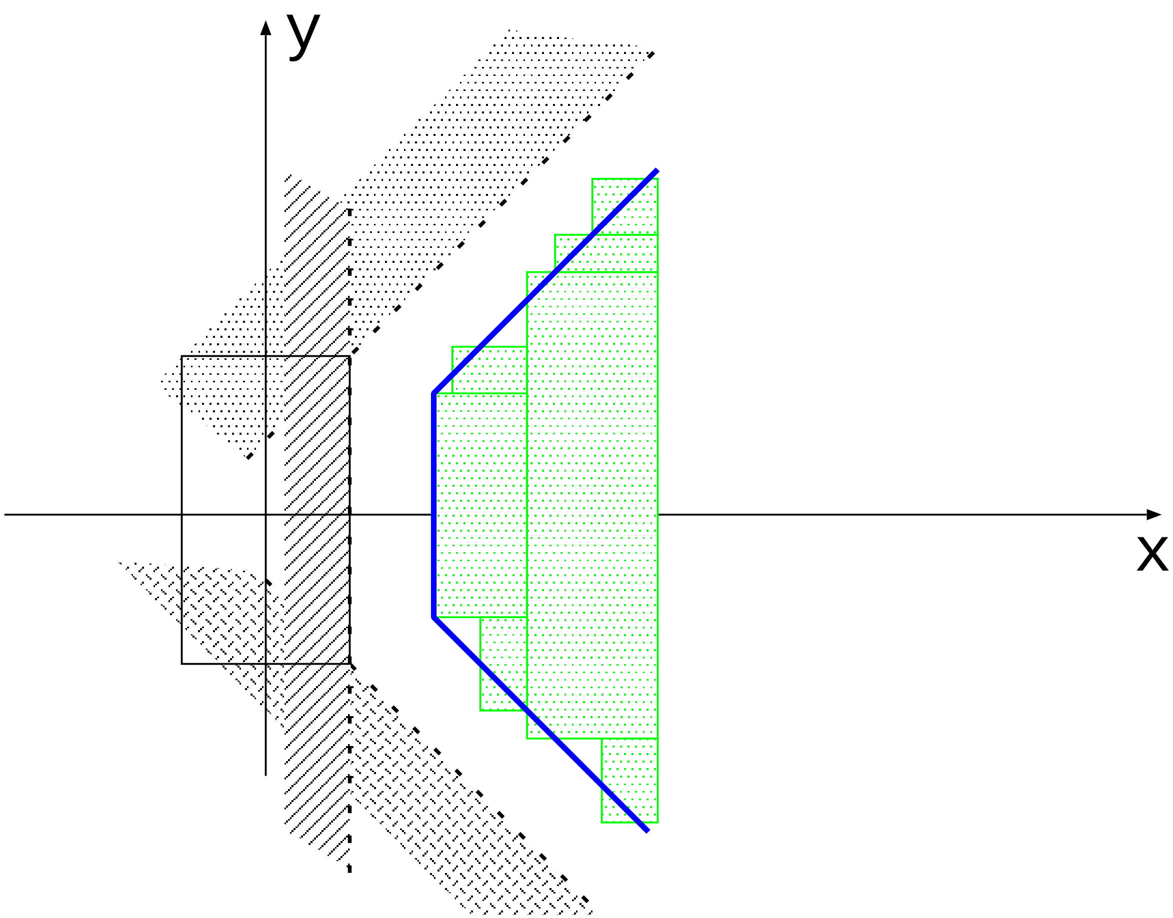
Machine Learning. Learning.

(1) (2) (3)



Resulting CAL5 decision tree.

Machine Learning. Learning.



"Pruned" decision tree.

Step 2

- Solving the constraints: generating depictions.

Machine Learning. Depiction Generation.

Step 2

- Solving the constraints: generating depictions.
- Use results of the learning step.

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- Binary relations: always two objects.

Machine Learning. Depiction Generation.

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- Solving the constraints: generating depictions.
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- Binary relations: always two objects.
  - Both objects unknown.

Machine Learning. Depiction Generation.

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- Solving the constraints: generating depictions.
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  - Both objects unknown.
  - First object already placed.

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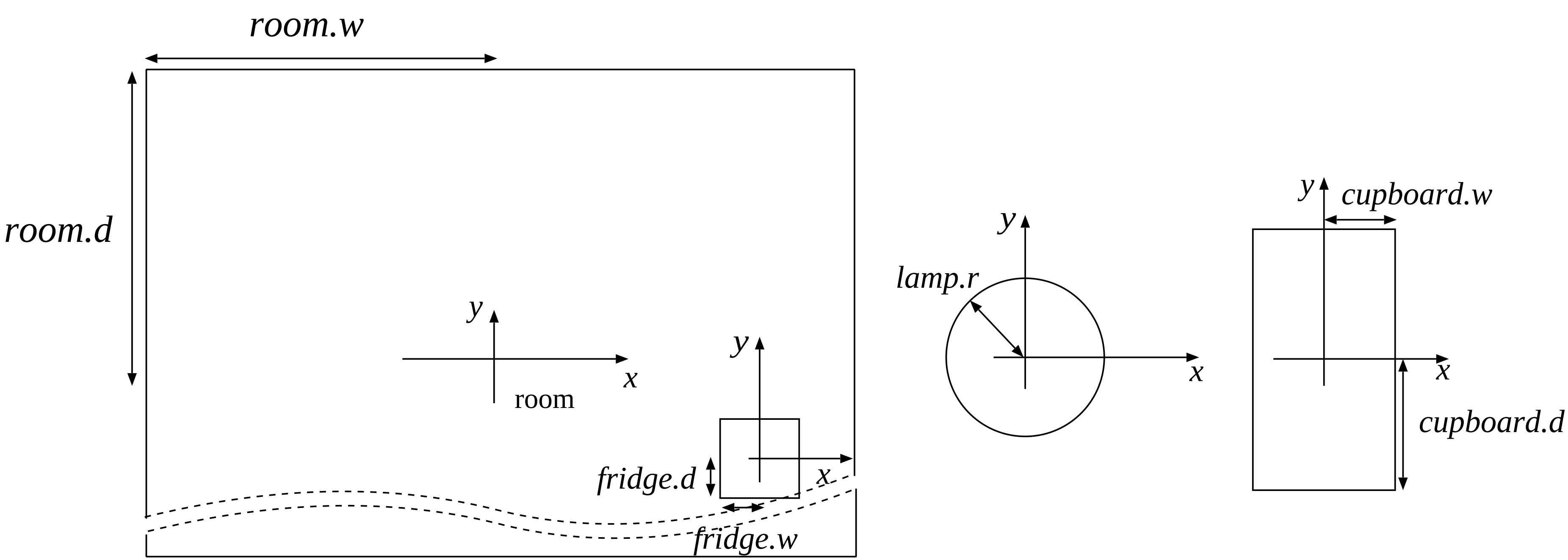
  - Both objects unknown.
  - First object already placed.
  - Second object placed.

Step 2

- Solving the constraints: generating depictions.
- Use results of the learning step.
- Binary relations: always two objects.
  - Both objects unknown.
  - First object already placed.
  - Second object placed.
  - Both objects already placed.

Machine Learning. Depiction Generation: 1.

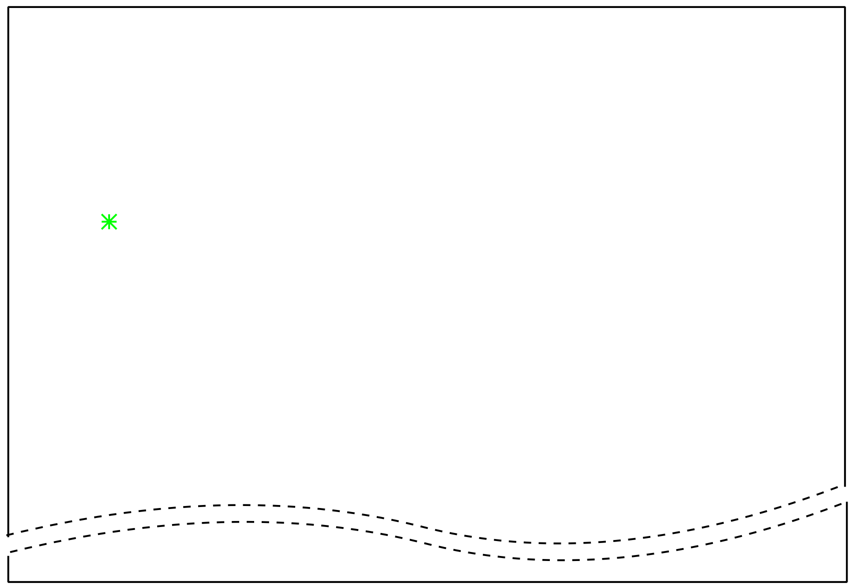


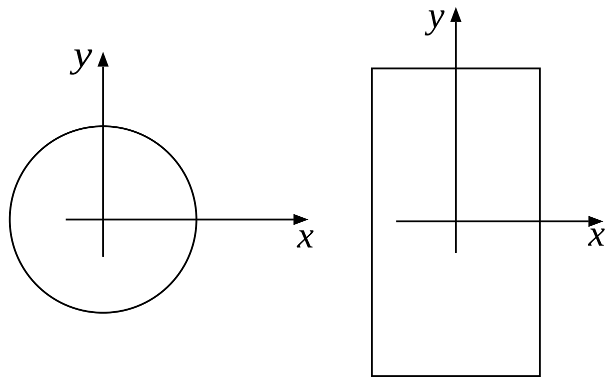
`right(cupboard, lamp)`

Both objects unknown.

Machine Learning. Depiction Generation: 1.

(1) (2) (3)

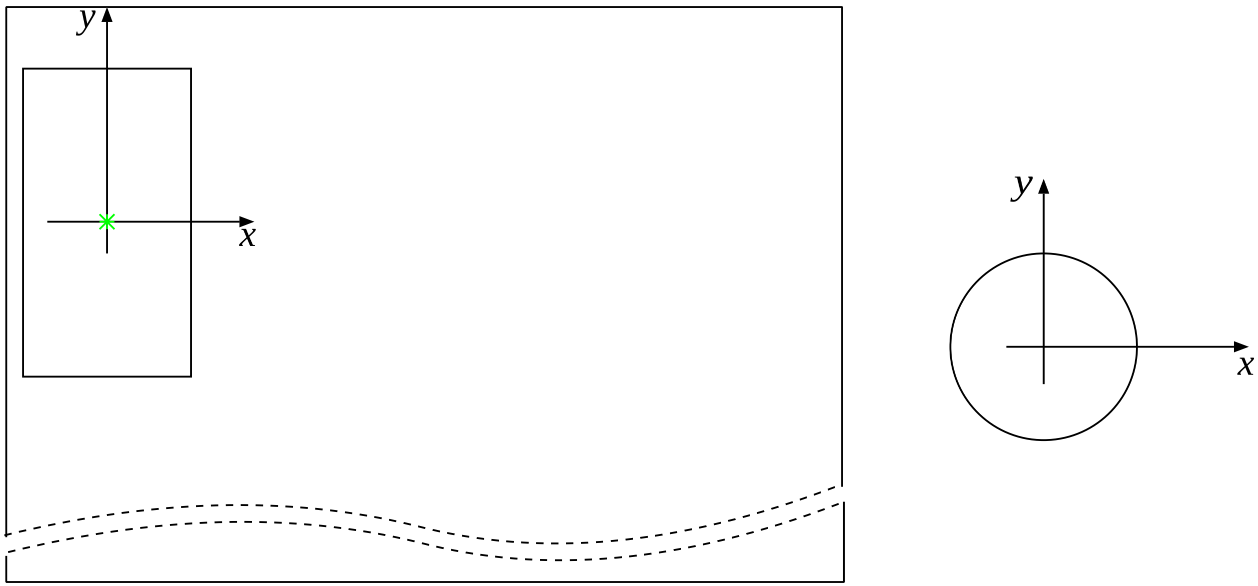




`right(cupboard, lamp)`

Select point randomly in room.

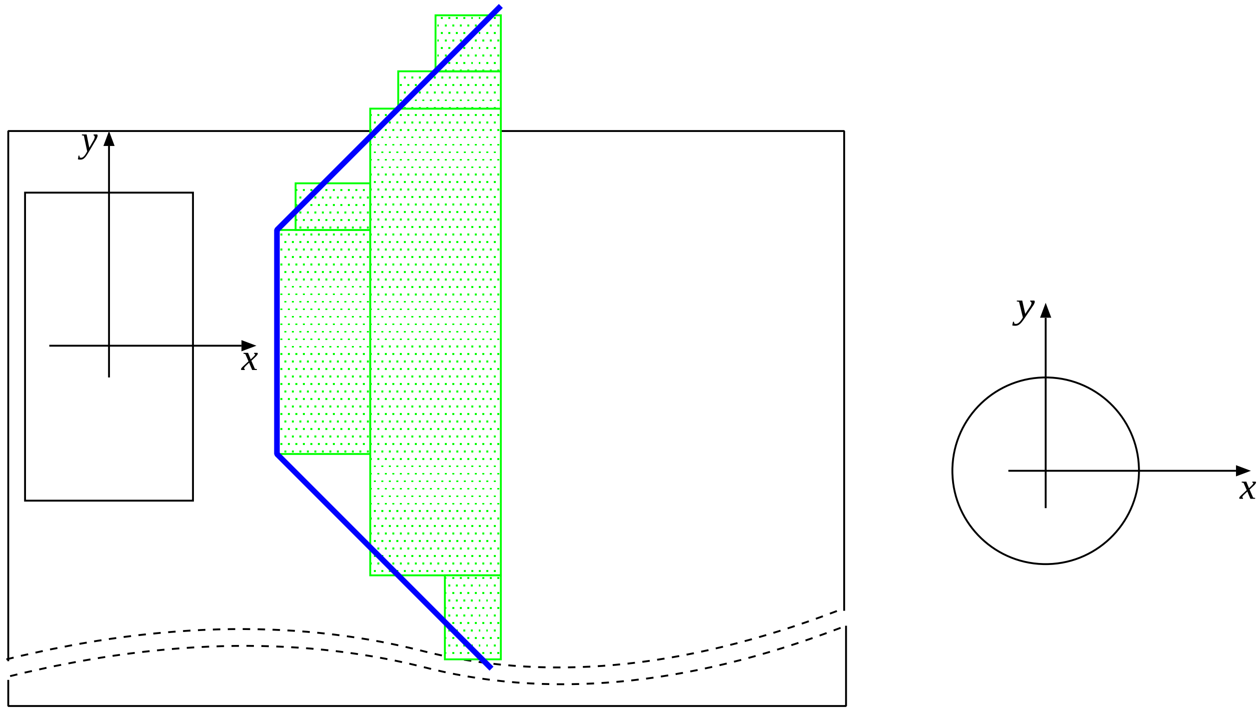
Machine Learning. Depiction Generation: 1.



`right(cupboard, lamp)`

Place cupboard.

Machine Learning. Depiction Generation: 1+2.

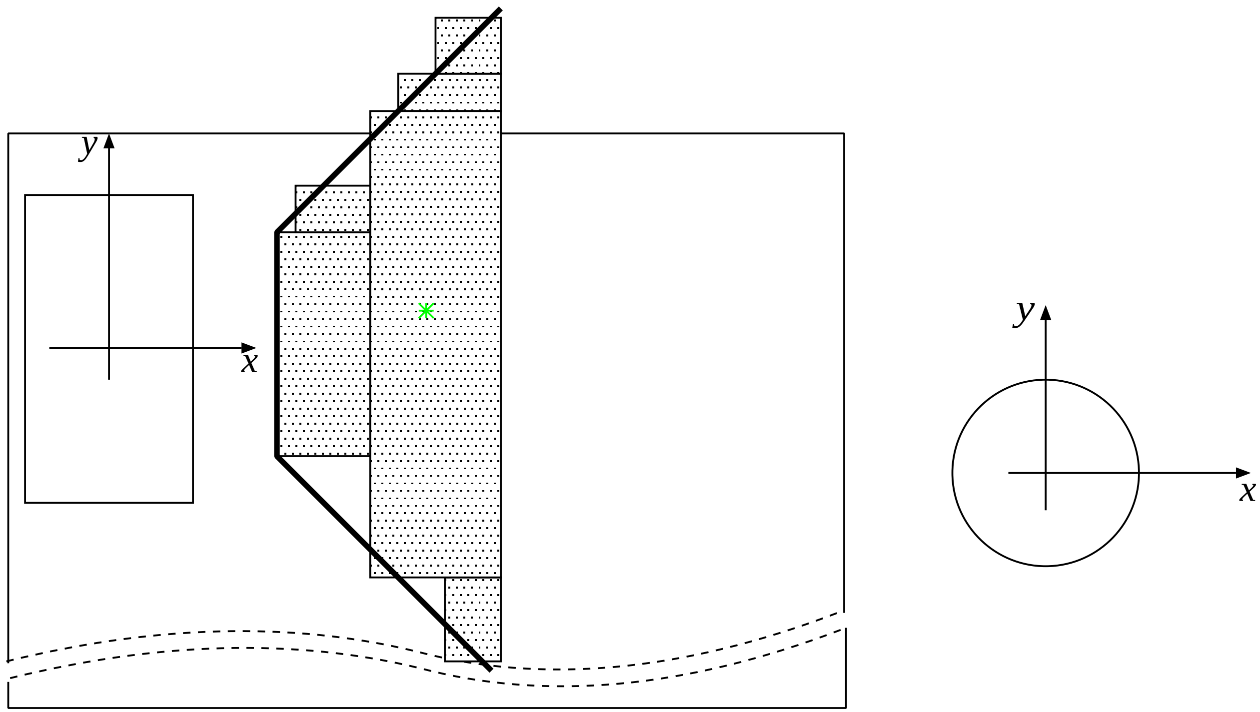


`right(cupboard, lamp)`

Use CAL5 decision tree.

Machine Learning. Depiction Generation: 1+2.

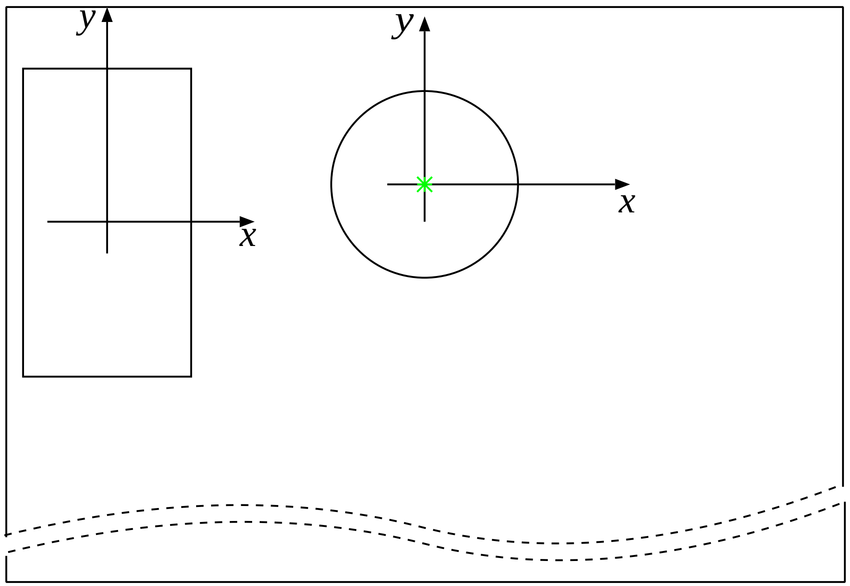
(1) (2) (3)



`right(cupboard, lamp)`

Select point in the class A region of the tree.

Machine Learning. Depiction Generation: 1+2.



`right(cupboard, lamp)`

Place lamp.

Spatial Inference – Constraint Solving and Learning

- Quantitative description of spatial relations.

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- Deciding spatial scenes using cooperative constraint solvers.

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- Machine learning: learning decision boundaries of spatial relations and generating depictions using the decision trees.

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Spatial Inference – Constraint Solving and Learning

- Quantitative description of spatial relations.
- Deciding spatial scenes using cooperative constraint solvers.
- Machine learning: learning decision boundaries of spatial relations and generating depictions using the decision trees.
- Cooperation of machine learning and constraint solving:
  - Detection of unsatisfiable descriptions using constraint solvers, fine-grain elaboration by learning approach.
  - Using constraint solvers for generating training sets.

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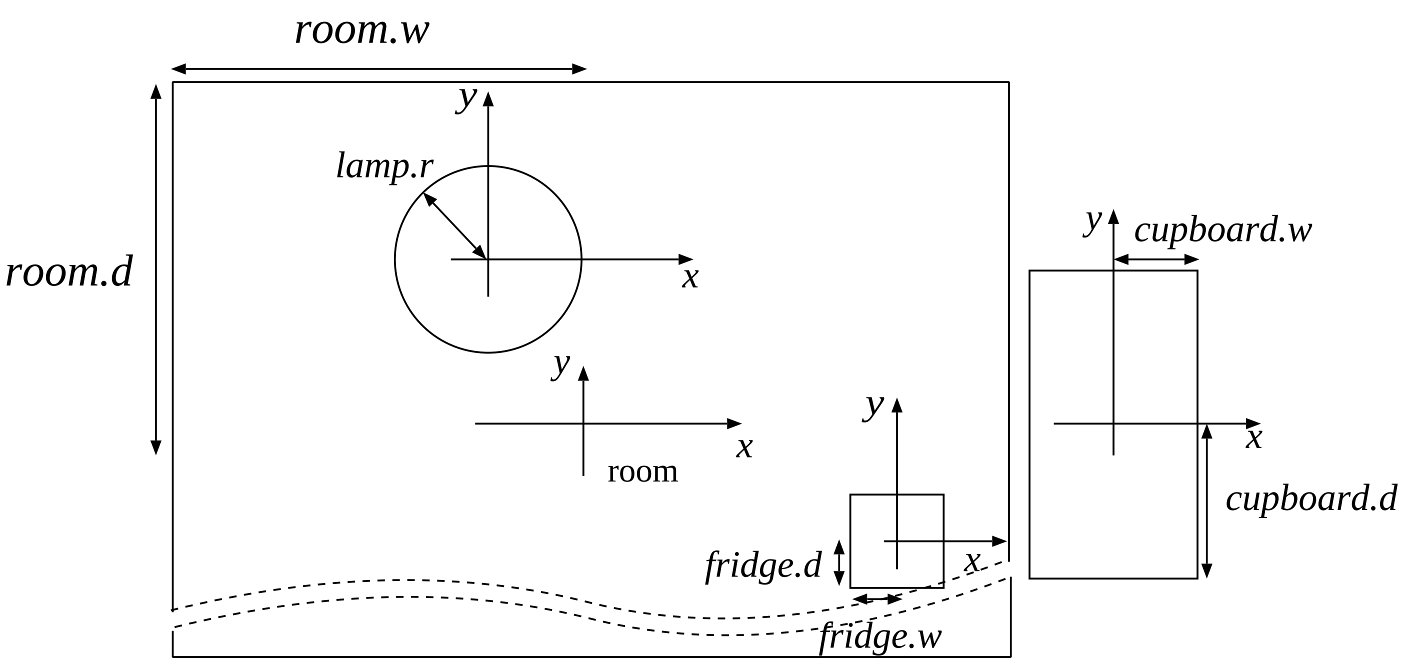
Machine Learning. Depiction Generation: 3.

(1) (2) (3)

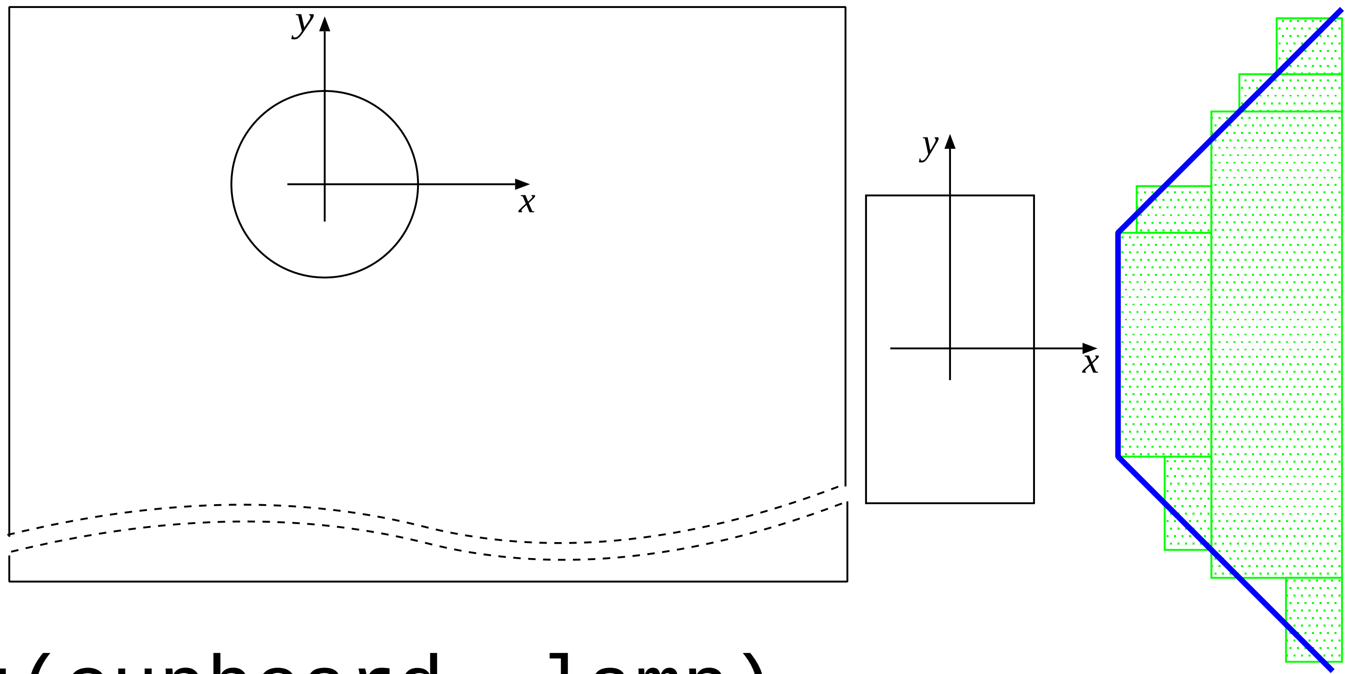


`right(cupboard, lamp)`

Lamp already placed.

Machine Learning. Depiction Generation: 3.

(1) (2) (3)

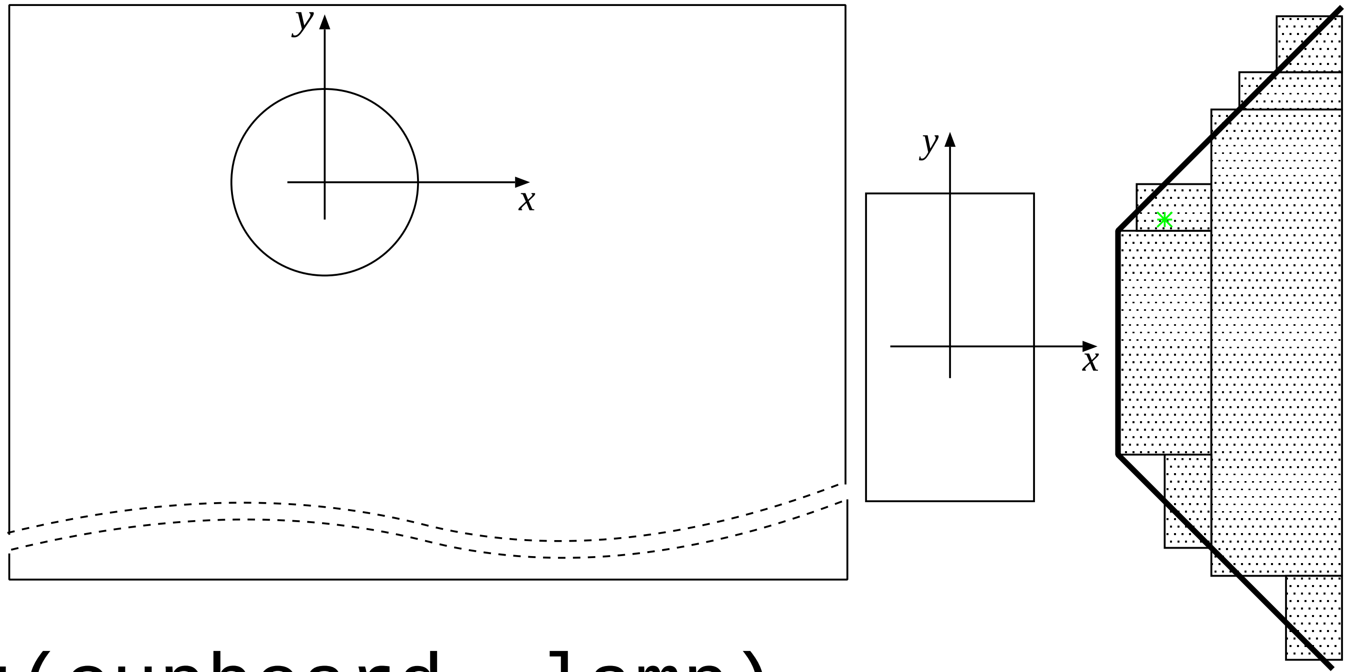


`right(cupboard, lamp)`

Use CAL5 decision tree.

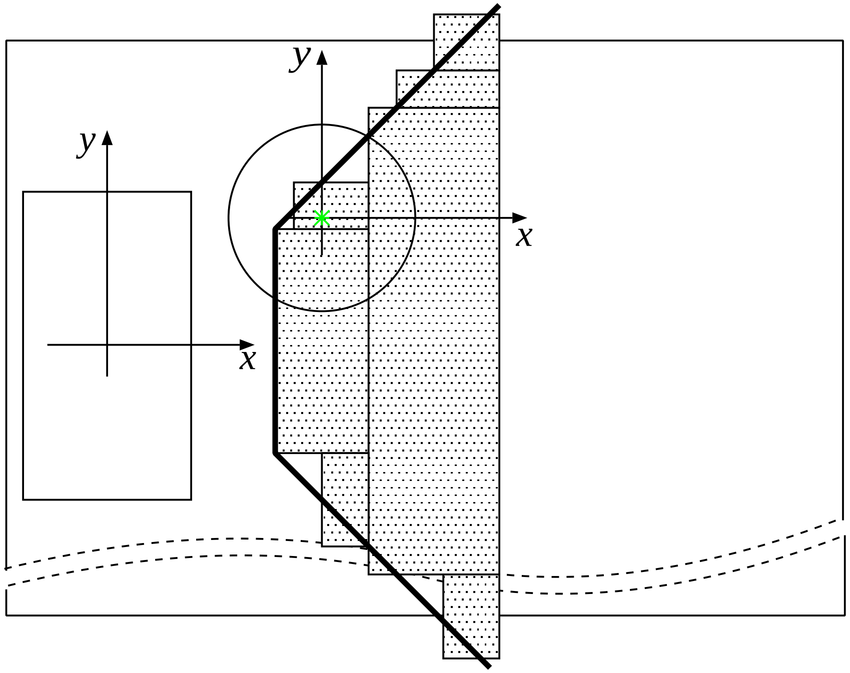
Machine Learning. Depiction Generation: 3.

(1) (2) (3)




```
right(cupboard, lamp)
```

Chose point in the class A region of the tree.

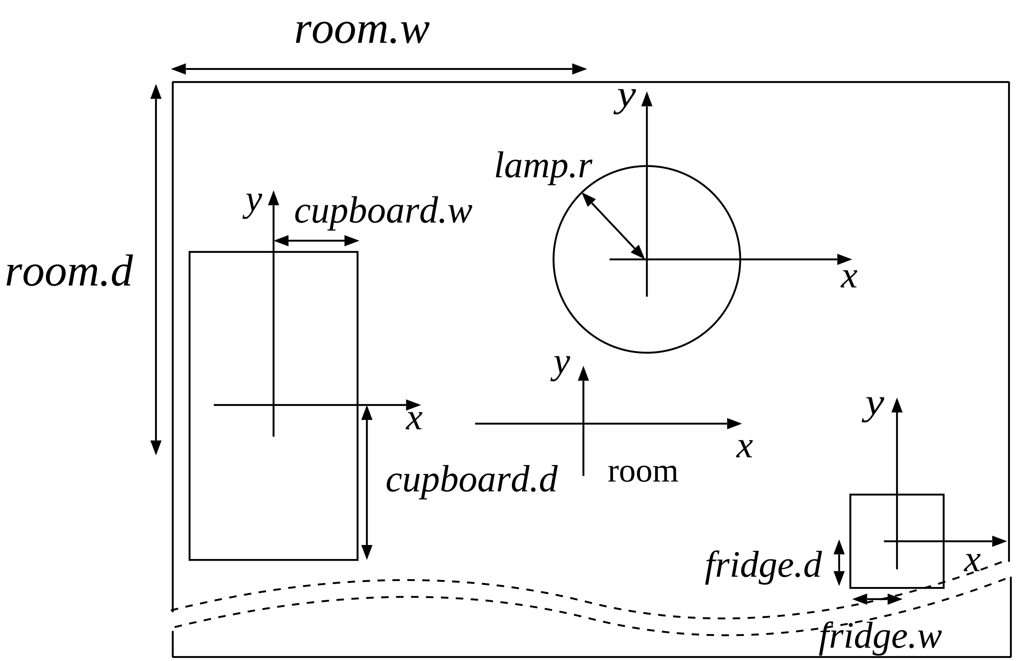


`right(cupboard, lamp)`

Map point to origin of lamp, backtransformation:
Cupboard fi xed.

Machine Learning. Depiction Generation: 4.

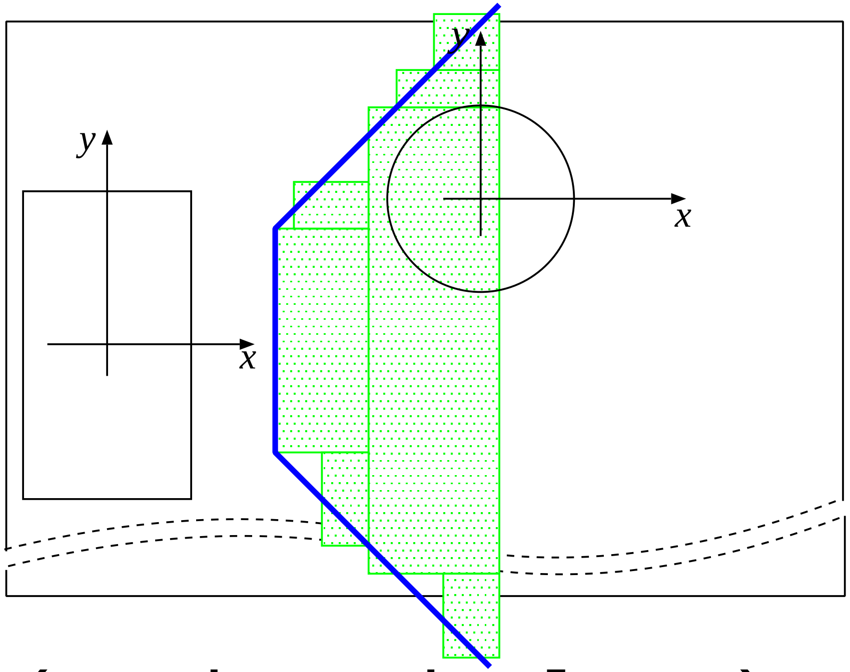
(1) (2) (3)



`right(cupboard, lamp)`

Both objects already placed.

Machine Learning. Depiction Generation: 4.

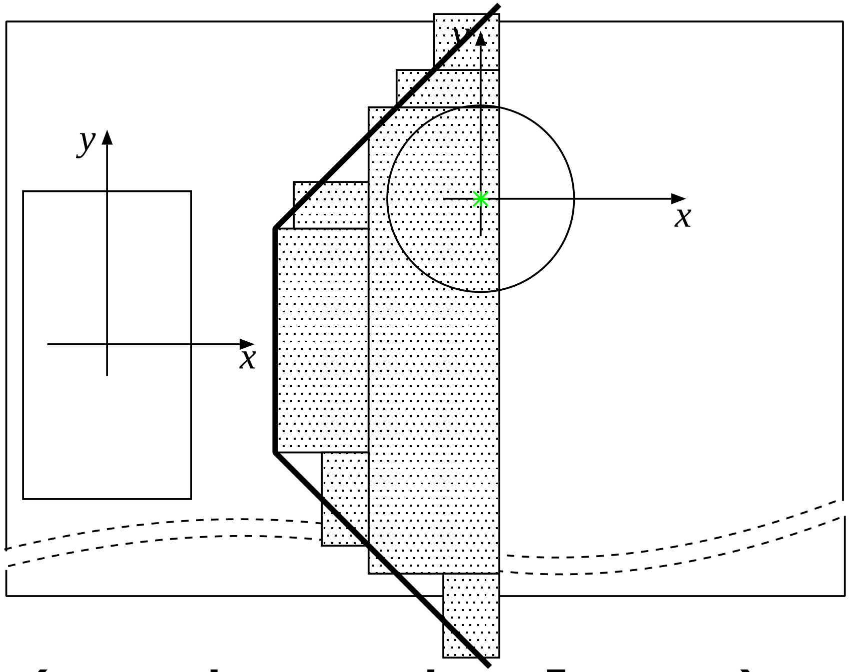


`right(cupboard, lamp)`

Use CAL5 decision tree.

Machine Learning. Depiction Generation: 4.

(1) (2) (3)



`right(cupboard, lamp)`

Origin of lamp is in the class A region of the tree: `right(cupboard, lamp)` holds.

Change of the basis:

$$\mathbf{p}_k{}^i = \mathbf{P}_j{}^i \mathbf{p}_k{}^j$$

$\mathbf{P}_j{}^i$ Homogeneous transformation matrix:

$$\mathbf{P}_j{}^i = \begin{pmatrix} \cos\Theta_j{}^i & -\sin\Theta_j{}^i & 0 & \Delta x_j{}^i \\ \sin\Theta_j{}^i & \cos\Theta_j{}^i & 0 & \Delta y_j{}^i \\ 0 & 0 & 1 & \Delta z_j{}^i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$