
Spatial Inference and Constraint Solving

How to Depict Textual Spatial Descriptions from Internet.

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Spatial Relations

Machine Learning

Generating Depictions

- Internet today: Still many text based applications, e.g. email, news.
- Starting point: Textual descriptions of spatial scenes.
- Aims:
 - Checking the texts for consistency.
 - Generating appropriate depictions.
 - Inferring new information (not initially given in the texts).
- Here: Consider only depiction generation.

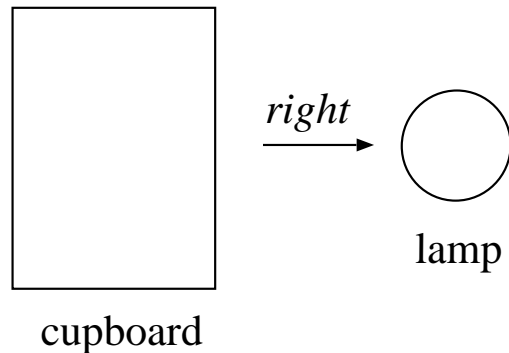
Introduction. Example Text.

(1) (2) (3)

Yesterday I got a nice new lamp for my kitchen. It stands right of the cupboard and on the left side of my fridge. Additionally, the lamp is left of the table.

Introduction. Example Text.

(1) (2) (3)

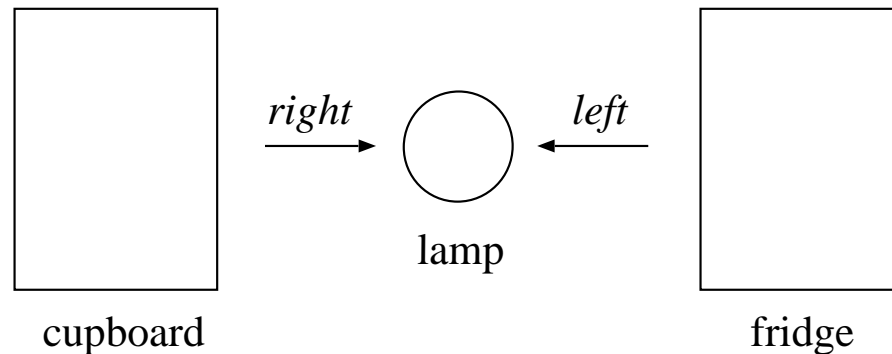


Yesterday I got a nice new **lamp** for my kitchen. It stands **right** of the **cupboard** and on the left side of my fridge. Additionally, the lamp is left of the table.

`right (cupboard, lamp)`

Introduction. Example Text.

(1) (2) (3)

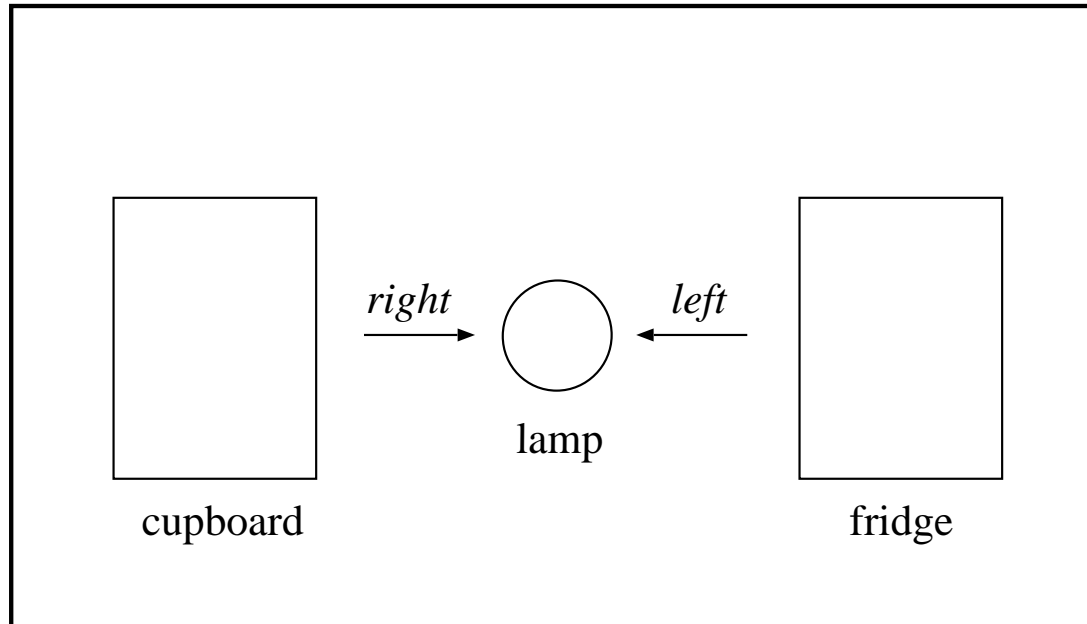


Yesterday I got a nice new **lamp** for my kitchen. It stands right of the cupboard and on the **left** side of my **fridge**. Additionally, the lamp is left of the table.

```
right(cupboard, lamp)
left(fridge, lamp)
```

Introduction. Example Text.

(1) (2) (3)



Yesterday I got a nice new lamp for my **kitchen**. It stands right of the cupboard and on the left side of my fridge. Additionally, the lamp is left of the table.

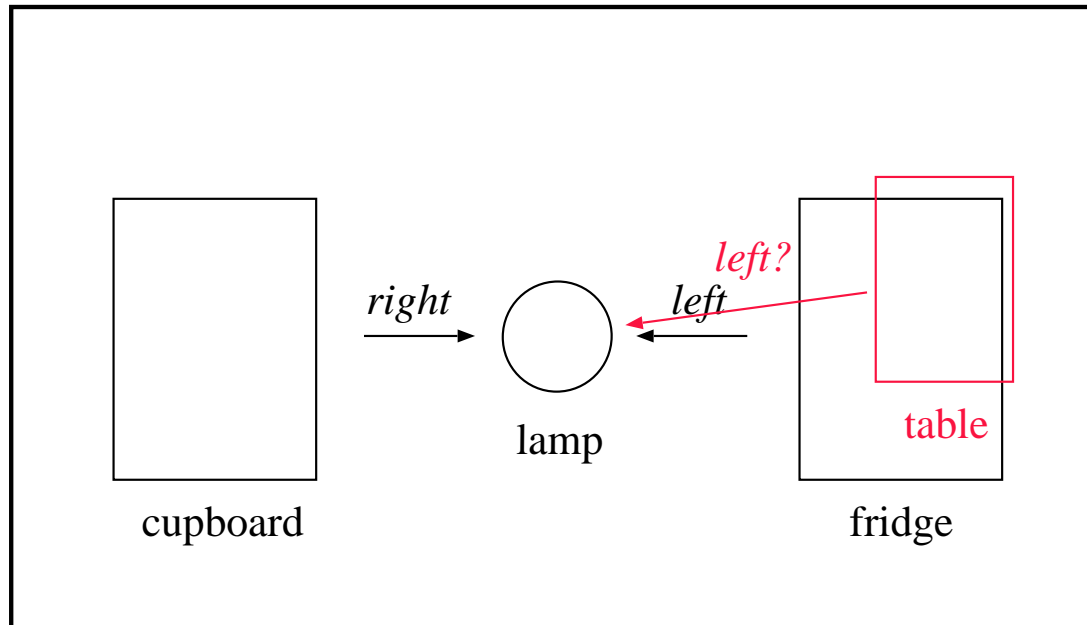
```
right(cupboard, lamp)
```

```
left(fridge, lamp)
```

```
in_room(fridge), in_room(lamp), in_room(cupboard)
```

Introduction. Example Text.

(1) (2) (3)



Yesterday I got a nice new lamp for my kitchen. It stands right of the cupboard and on the left side of my fridge. Additionally, the **lamp** is **left** of the **table**.

`right(cupboard, lamp)`

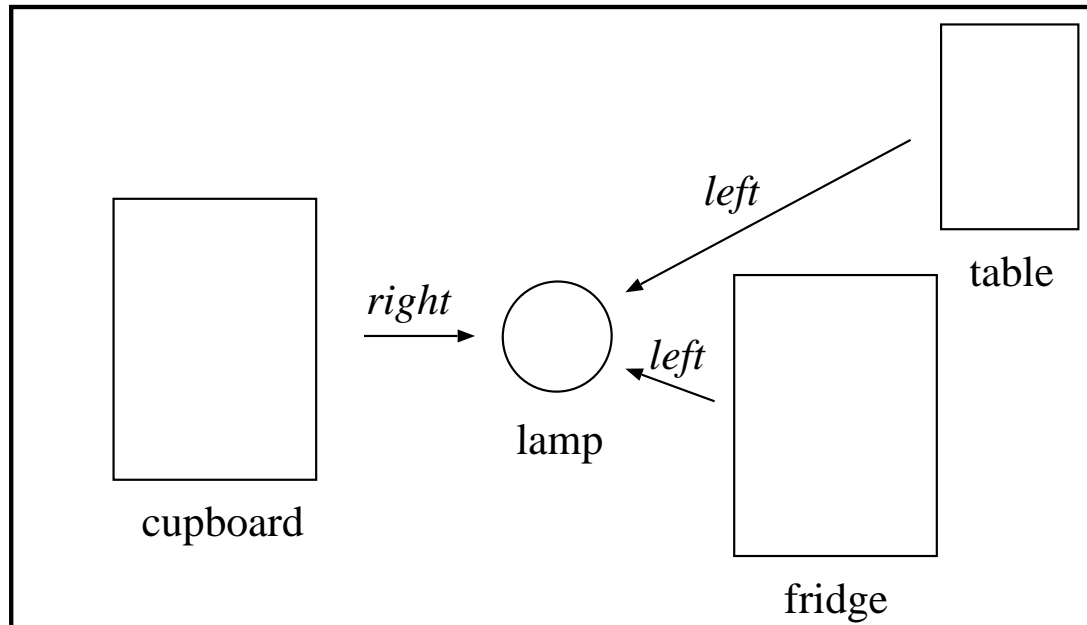
`left(fridge, lamp)`

`in_room(fridge), in_room(lamp), in_room(cupboard)`

`left(table, lamp)?`

Introduction. Example Text.

(1) (2) (3)



Yesterday I got a nice new lamp for my kitchen. It stands right of the cupboard and on the left side of my fridge. Additionally, the lamp is left of the table.

```
right(cupboard, lamp)
```

```
left(fridge, lamp)
```

```
in_room(fridge), in_room(lamp), in_room(cupboard)
```

```
left(table, lamp)
```

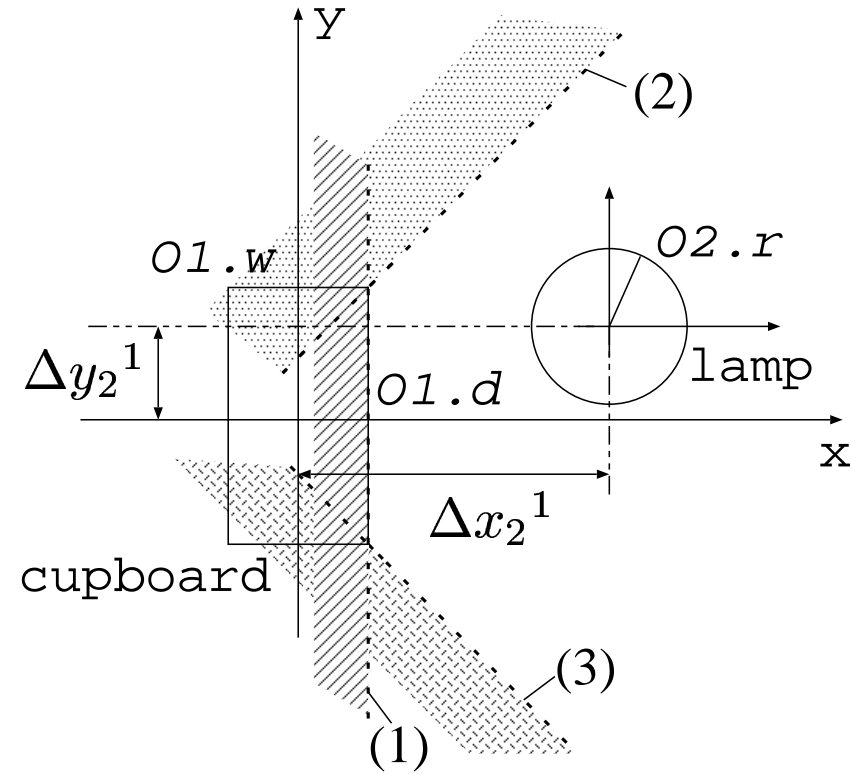
Outline

- Definition of spatial relations.
- Machine learning approach:
 - Learning of relations.
 - Generating depictions.

Spatial Relations. A Metric Approach.

(1) (2) (3)

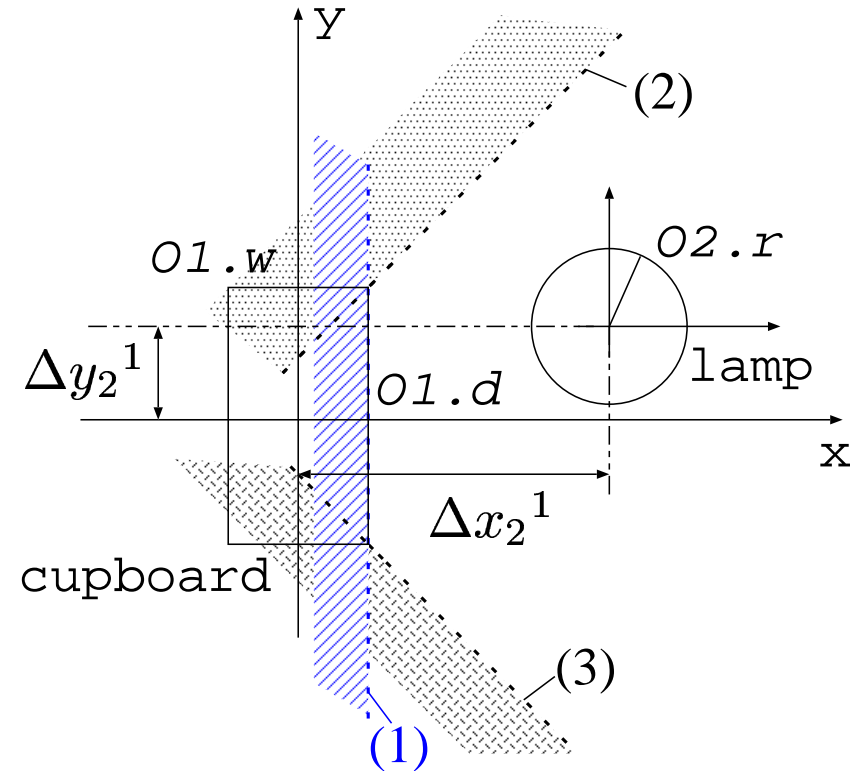
right(cupboard, lamp)



Spatial Relations. A Metric Approach.

(1) (2) (3)

right(cupboard, lamp)



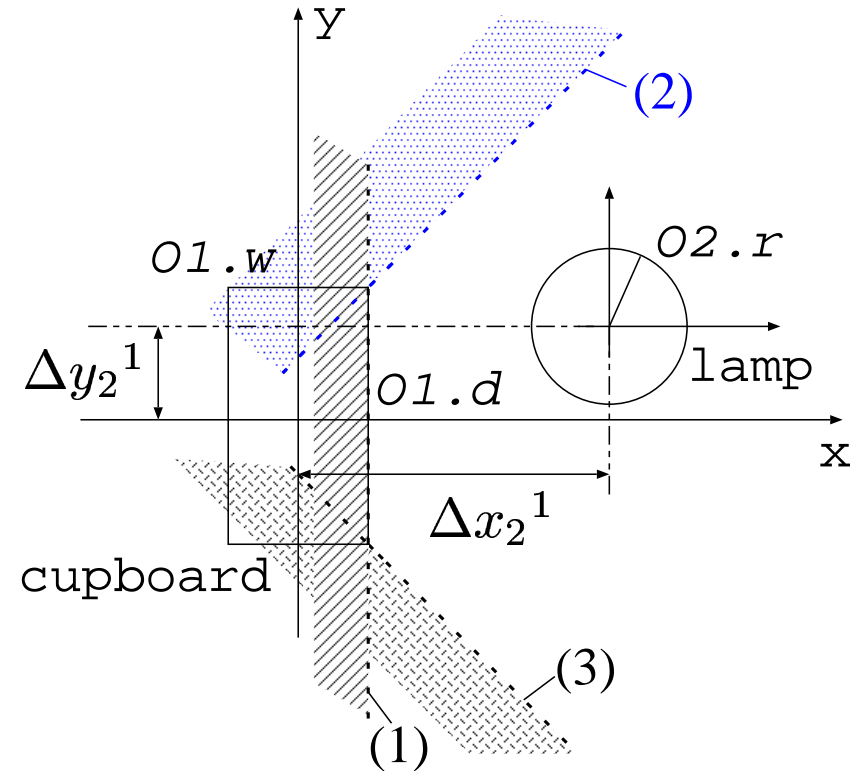
Mathematical description: $\text{right}(O1, O2)$

$$\Delta x_2^1 \geq O1.w + O2.r \quad (1)$$

Spatial Relations. A Metric Approach.

(1) (2) (3)

right (cupboard, lamp)



Mathematical description: right (O1, O2)

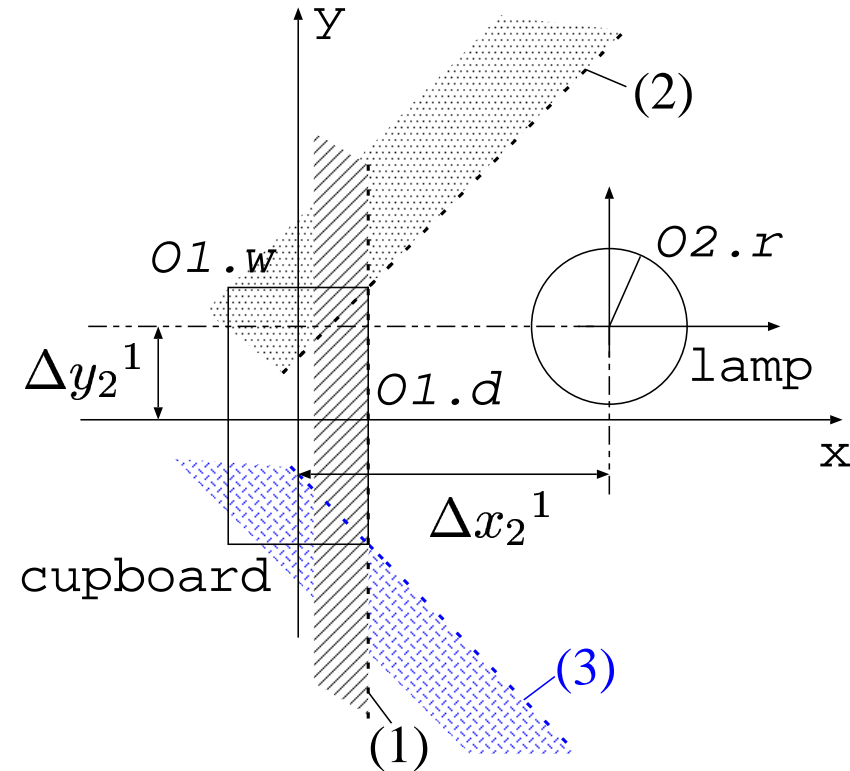
$$\Delta x_2^1 \geq O1.w + O2.r \quad (1)$$

$$\Delta x_2^1 \geq \Delta y_2^1 + O1.w - O1.d + \sqrt{2}O2.r \quad (2)$$

Spatial Relations. A Metric Approach.

(1) (2) (3)

right(cupboard, lamp)



Mathematical description: right($O1$, $O2$)

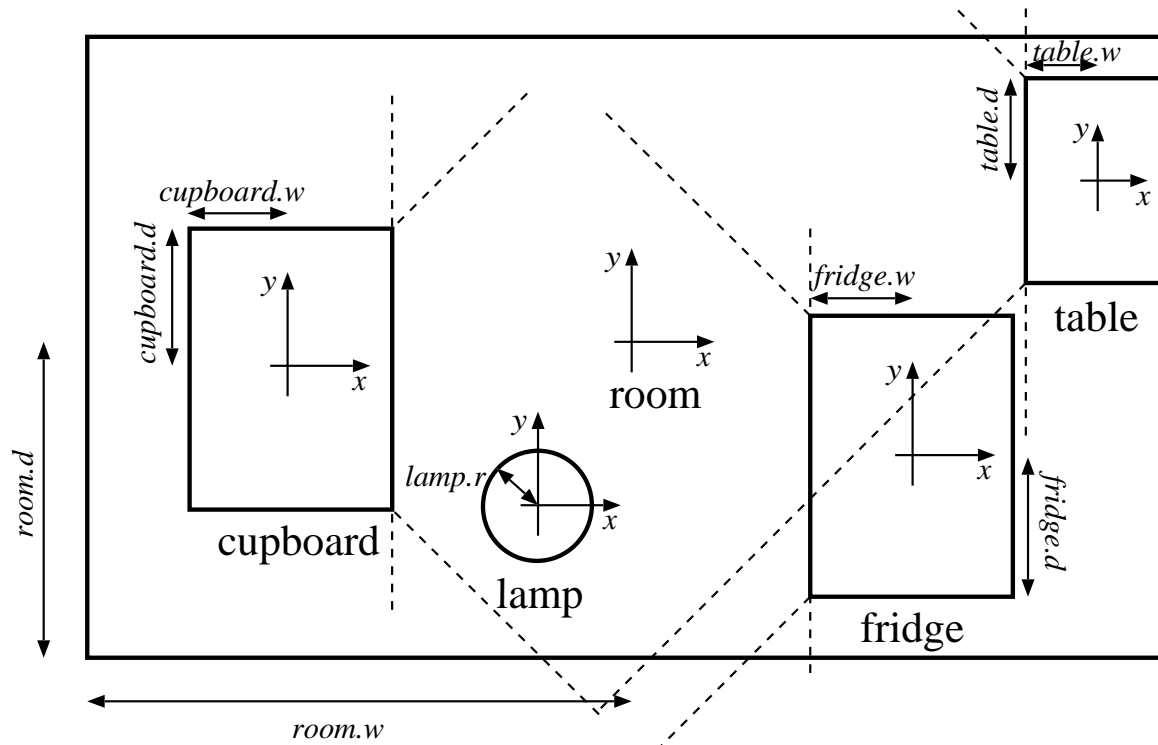
$$\Delta x_2^1 \geq O1.w + O2.r \quad (1)$$

$$\Delta x_2^1 \geq \Delta y_2^1 + O1.w - O1.d + \sqrt{2}O2.r \quad (2)$$

$$\Delta x_2^1 \geq -\Delta y_2^1 + O1.w - O1.d + \sqrt{2}O2.r \quad (3)$$

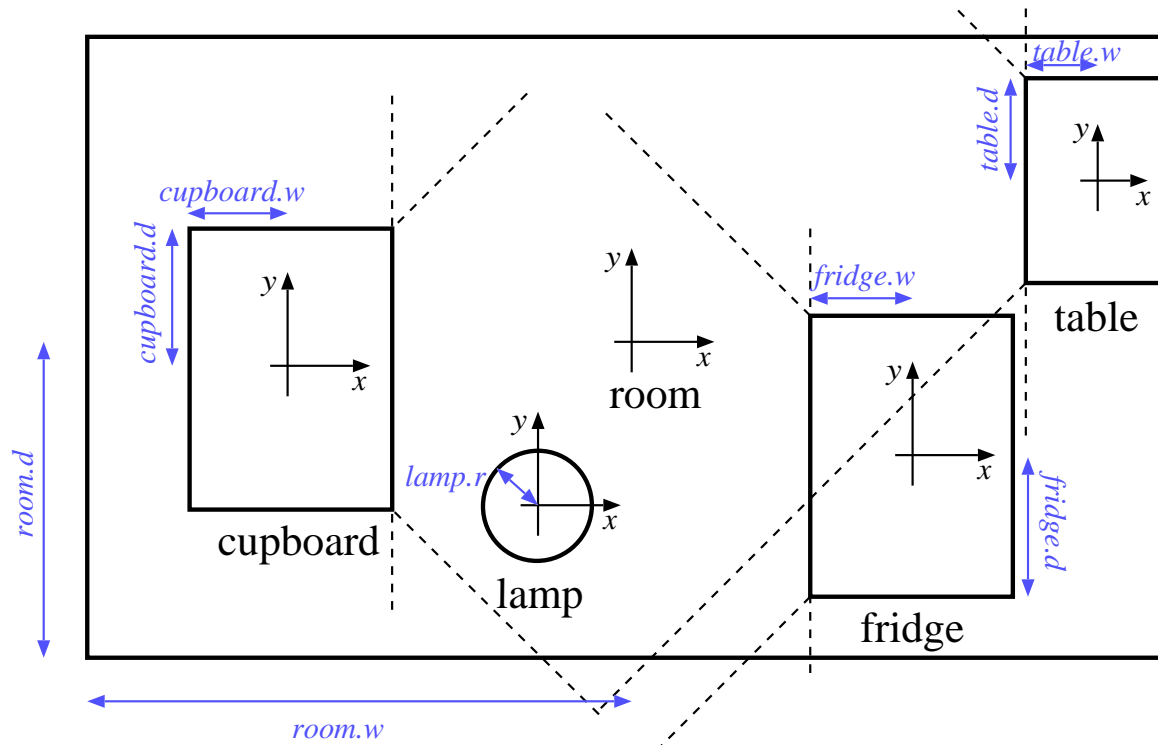
Spatial Relations. Example Scene Revisited.

(1) (2) (3)



Spatial Relations. Example Scene Revisited.

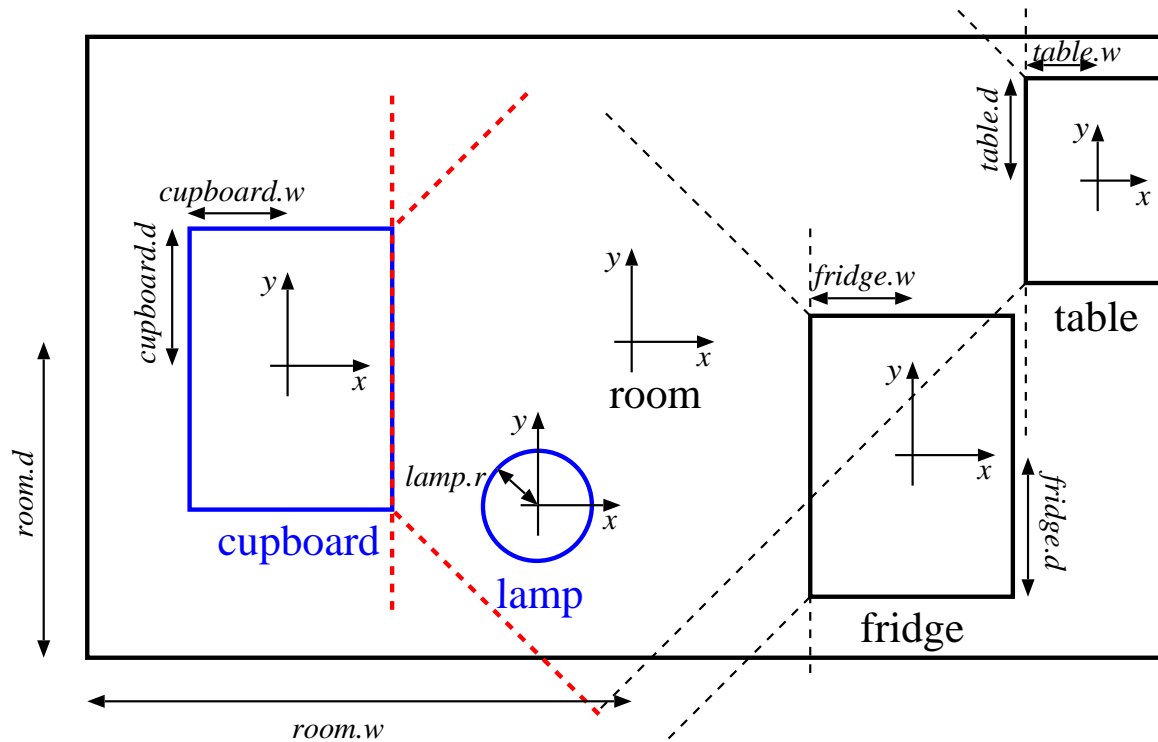
(1) (2) (3)



$room.w \in [4.0, 4.5]$, $room.d \in [4.0, 4.5]$,
 $fridge.w \in [0.4, 0.5]$, $fridge.d \in [0.4, 0.5]$,
 $cupboard.w = 0.4$, $cupboard.d \in [1.0, 1.2]$,
 $lamp.r = 0.3$

Spatial Relations. Example Scene Revisited.

(1) (2) (3)

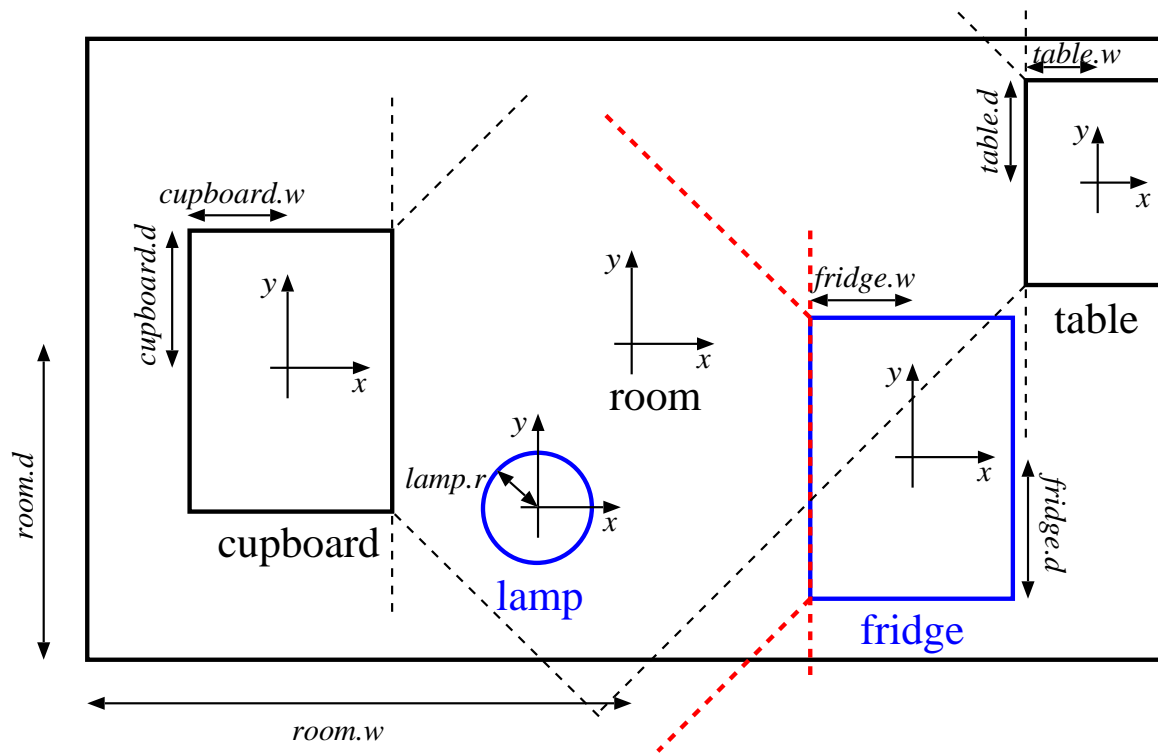


$room.w \in [4.0, 4.5]$, $room.d \in [4.0, 4.5]$,
 $fridge.w \in [0.4, 0.5]$, $fridge.d \in [0.4, 0.5]$,
 $cupboard.w = 0.4$, $cupboard.d \in [1.0, 1.2]$,
 $lamp.r = 0.3$

$right(cupboard, lamp)$

Spatial Relations. Example Scene Revisited.

(1) (2) (3)

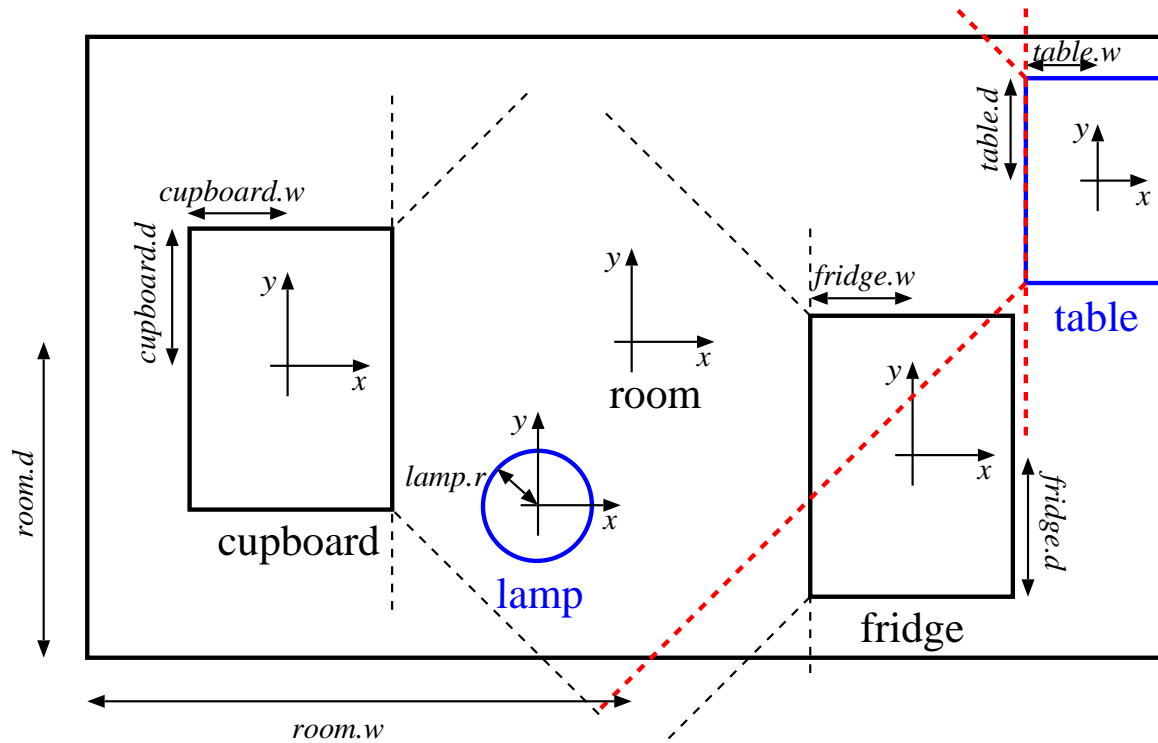


$room.w \in [4.0, 4.5]$, $room.d \in [4.0, 4.5]$,
 $fridge.w \in [0.4, 0.5]$, $fridge.d \in [0.4, 0.5]$,
 $cupboard.w = 0.4$, $cupboard.d \in [1.0, 1.2]$,
 $lamp.r = 0.3$

$right(cupboard, lamp)$
 $left(fridge, lamp)$

Spatial Relations. Example Scene Revisited.

(1) (2) (3)



$room.w \in [4.0, 4.5]$, $room.d \in [4.0, 4.5]$,
 $fridge.w \in [0.4, 0.5]$, $fridge.d \in [0.4, 0.5]$,
 $cupboard.w = 0.4$, $cupboard.d \in [1.0, 1.2]$,
 $lamp.r = 0.3$

$right(cupboard, lamp)$
 $left(fridge, lamp)$
 $left(table, lamp)$

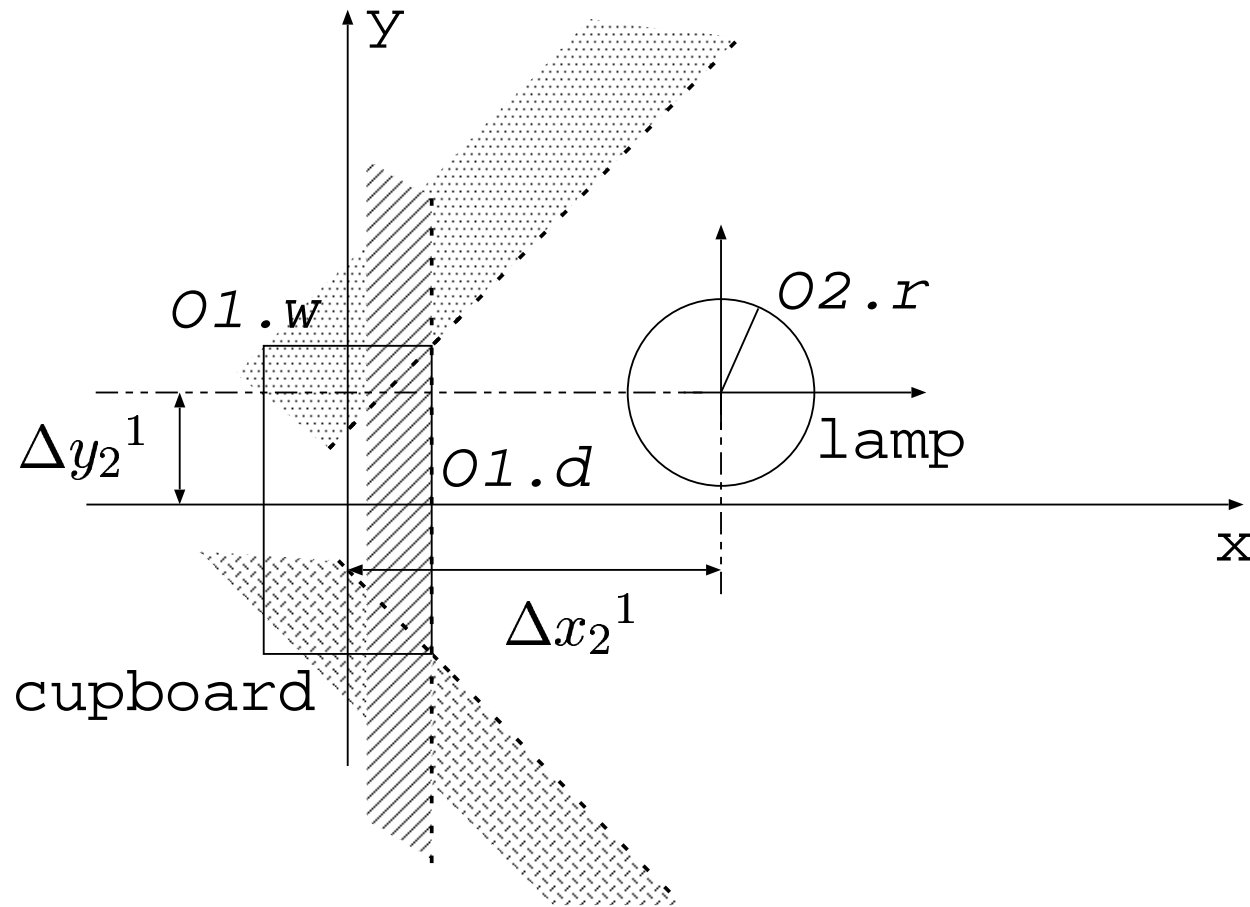
- Robotics: change reference system:

$$\mathbf{p}_k^i = \mathbf{P}_j^i \mathbf{p}_k^j$$

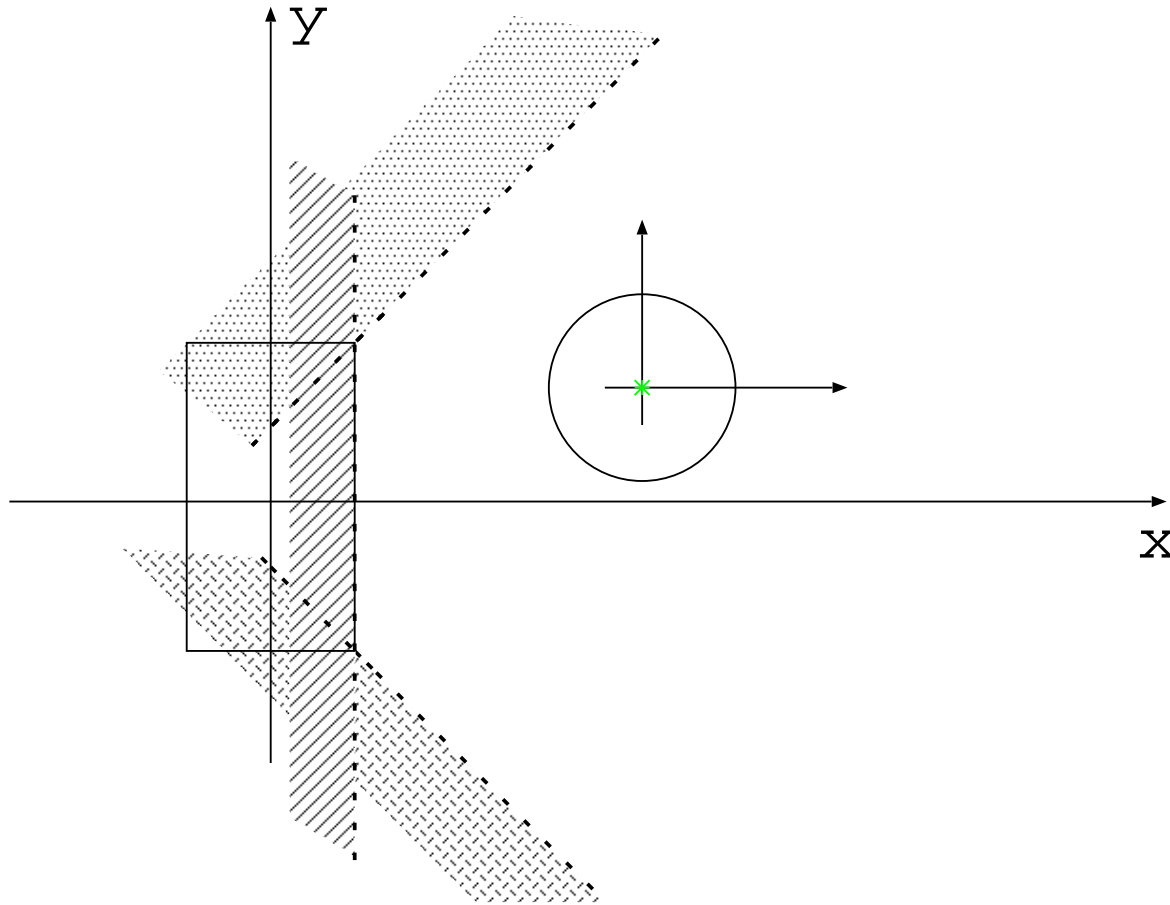
- \mathbf{P}_j^i Homogeneous transformation matrix:

$$\mathbf{P}_j^i = \begin{pmatrix} \cos \theta_j^i & -\sin \theta_j^i & 0 & \Delta x_j^i \\ \sin \theta_j^i & \cos \theta_j^i & 0 & \Delta y_j^i \\ 0 & 0 & 1 & \Delta z_j^i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

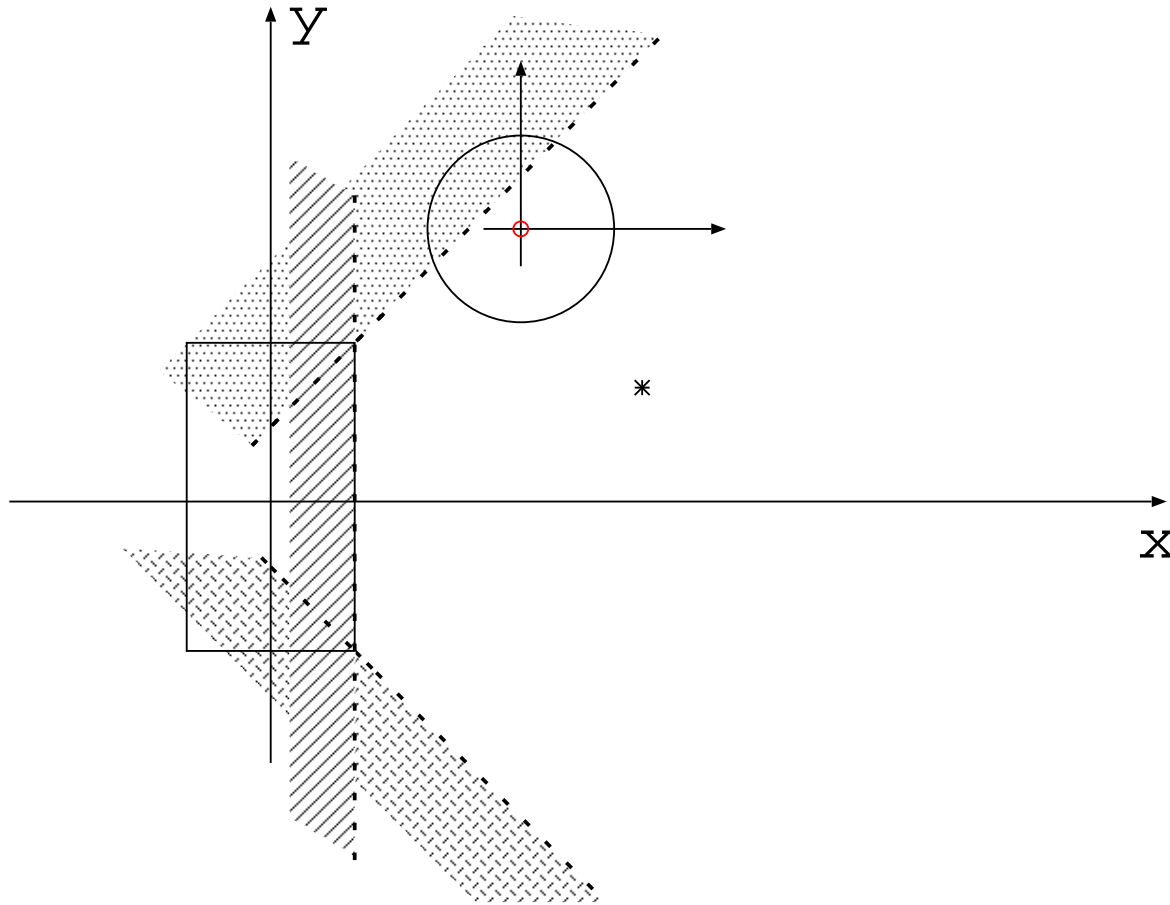
- Solving constraints for generating depictions.
- Constraints: systems of inequations containing trigonometric functions.
- Constraint solver are often incomplete.
- 2-Step machine learning algorithm:
 - Learning the problem, i.e. the decision boundary.
 - Use results for generating depictions.



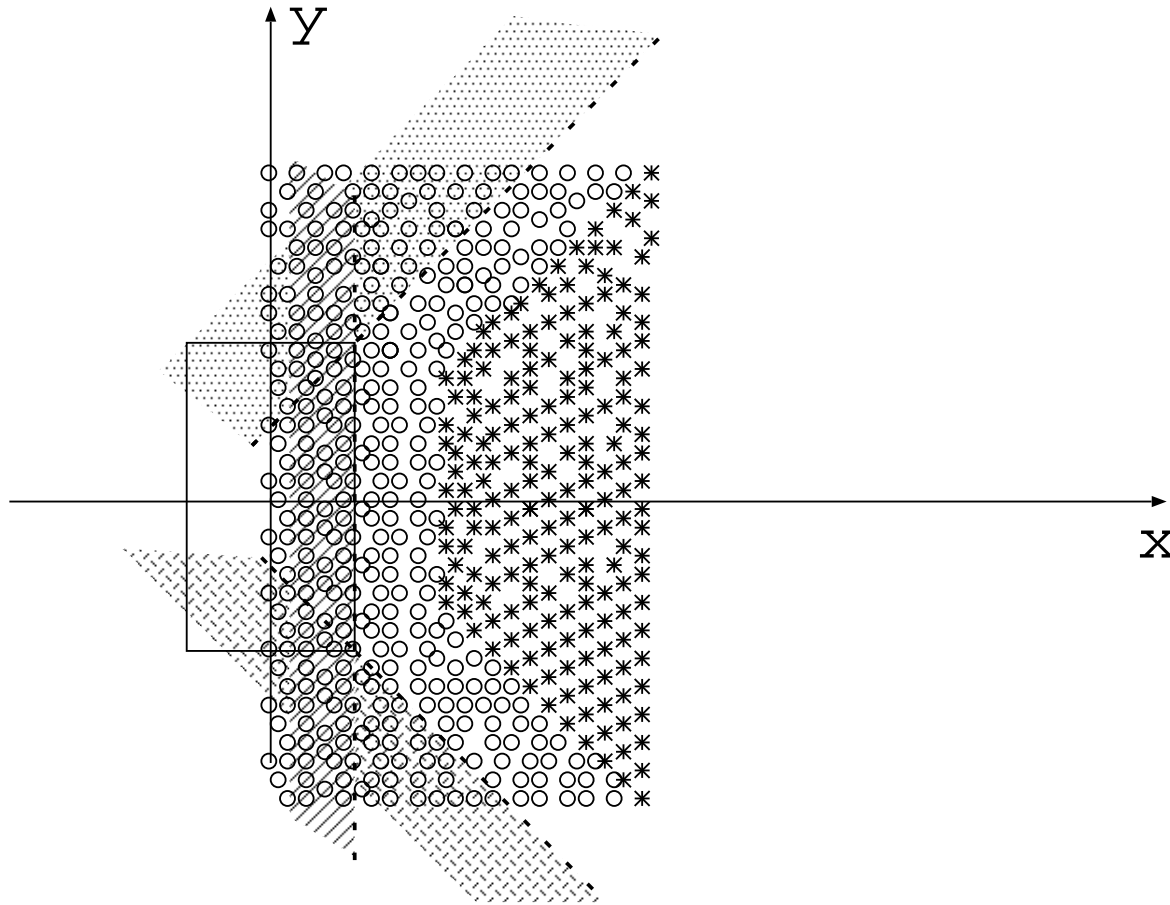
right (cupboard, lamp).



Slice of configuration space: parameters w , d , r clamped.



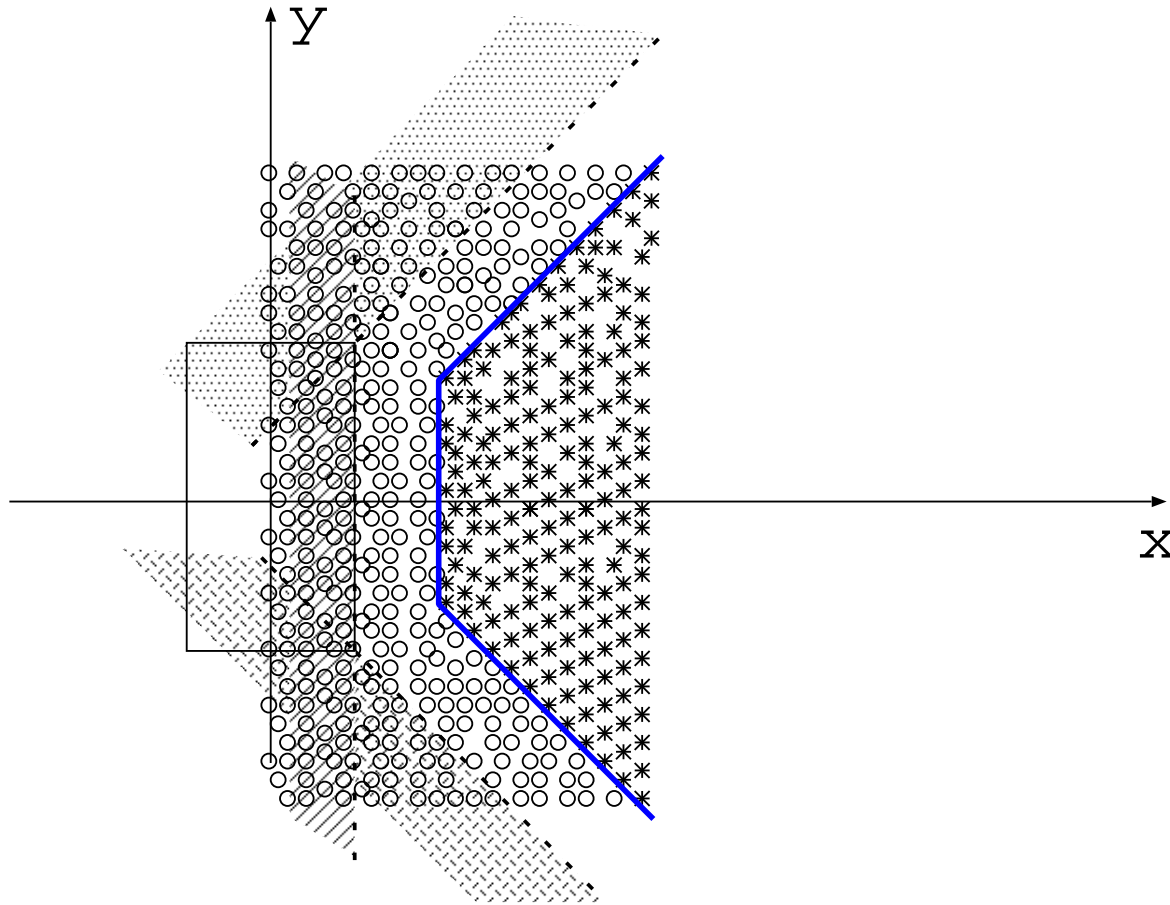
Slice of configuration space: parameters w , d , r clamped.



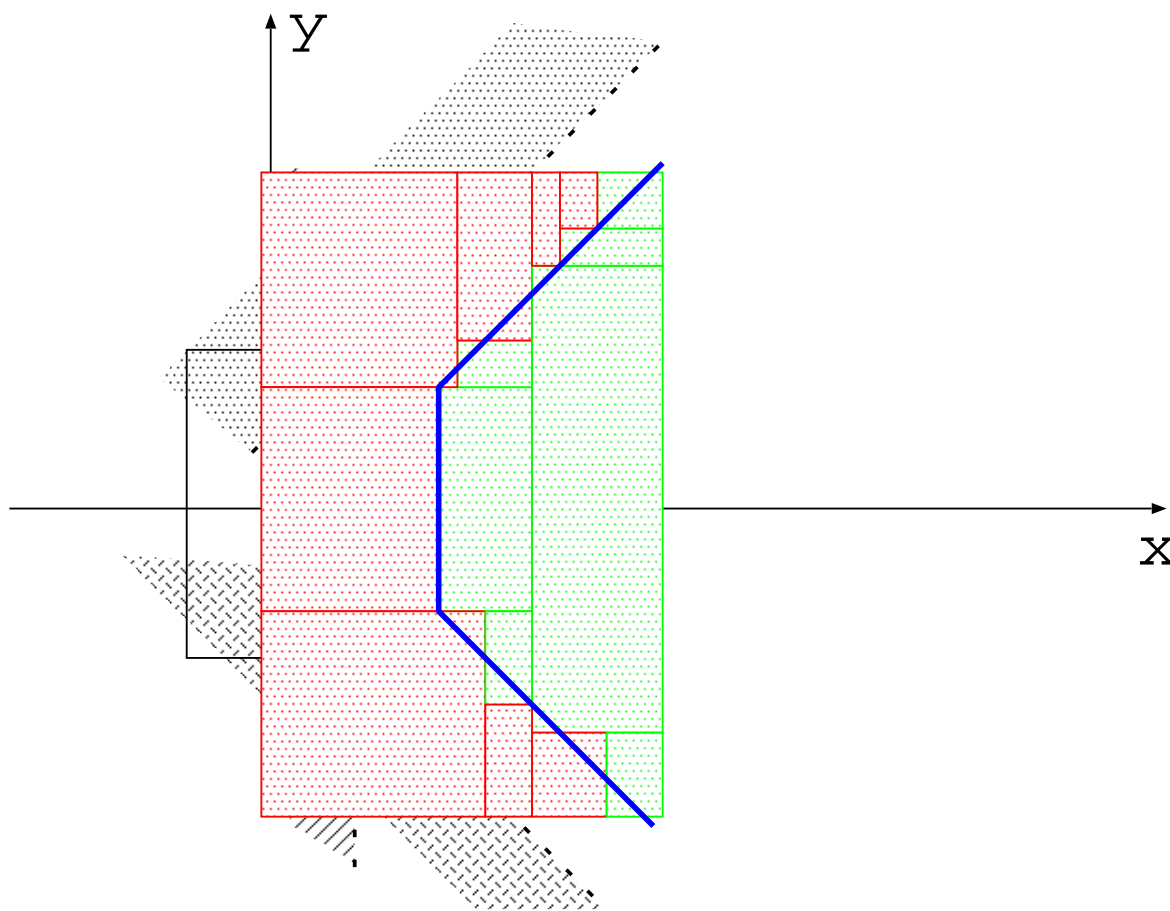
Dataset for training with CAL5.

Machine Learning. Learning the Spatial Relations.

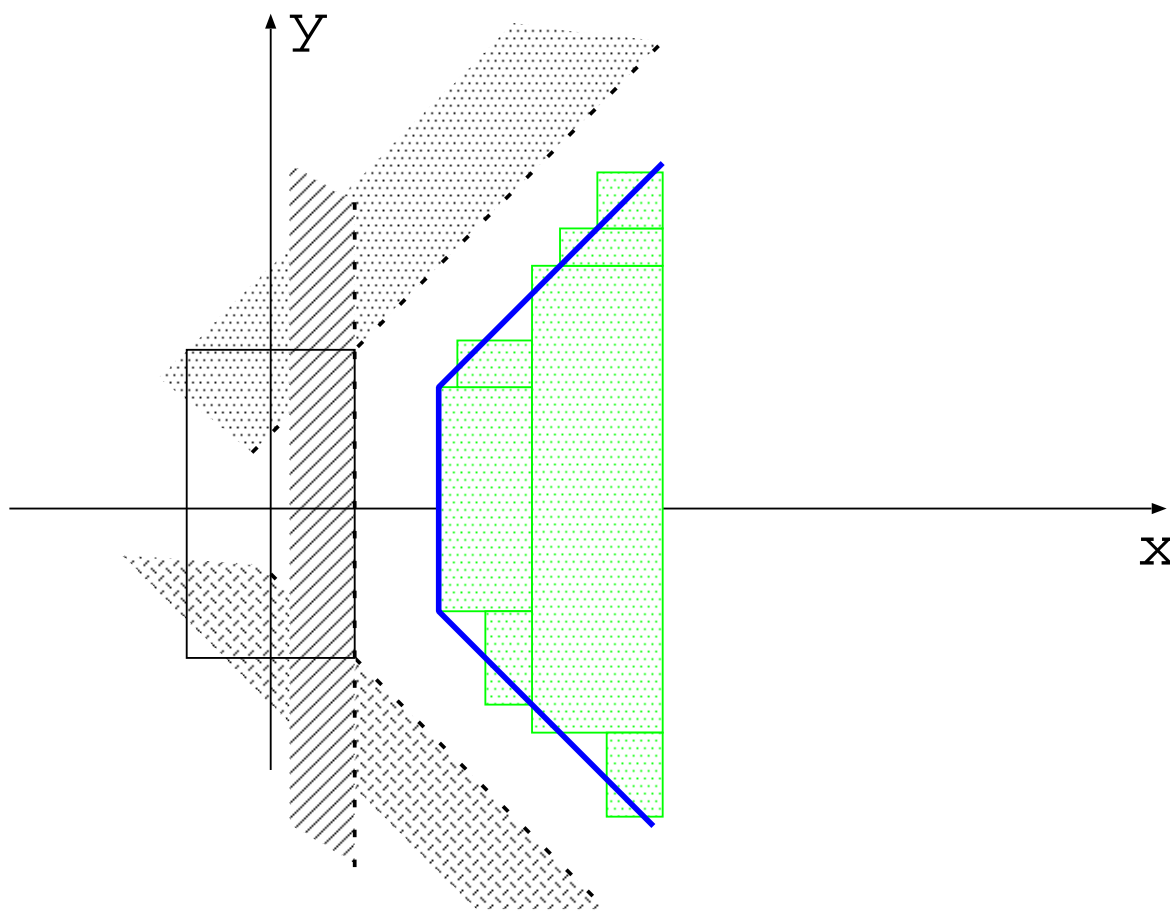
(1) (2) (3)



Decision boundary (to be learned).



Resulting CAL5 decision tree.

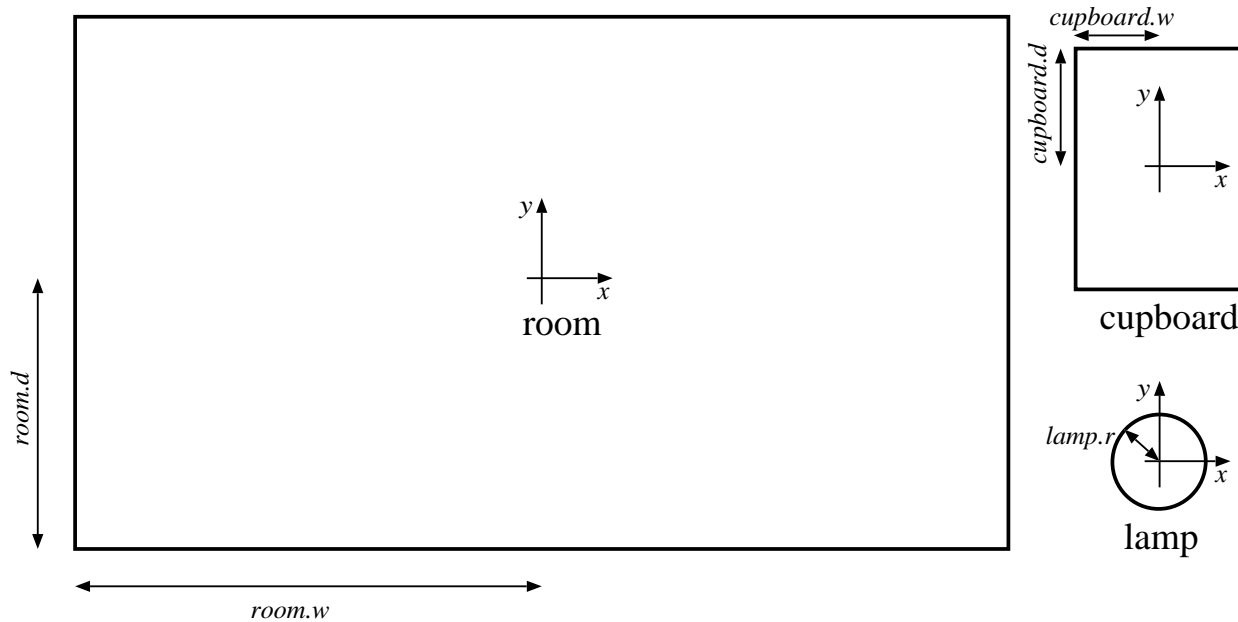


"Pruned" decision tree.

-
- Solving the constraints: generating depictions.
 - Use results of the learning step.
 - Binary relations: always two objects.
 - Both objects unknown.
 - First object already placed.
 - Second object placed.
 - Both objects already placed.

Generating Depictions. Case 1.

(1) (2) (3)

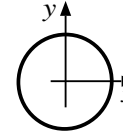
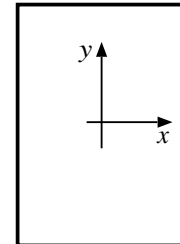
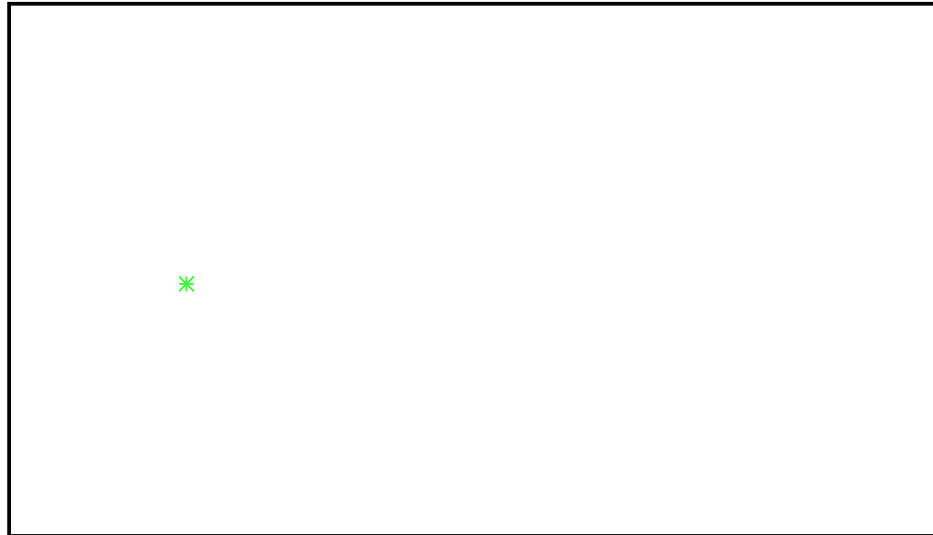


`right (cupboard, lamp)`

Both objects unknown.

Generating Depictions. Case 1.

(1) (2) (3)

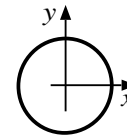
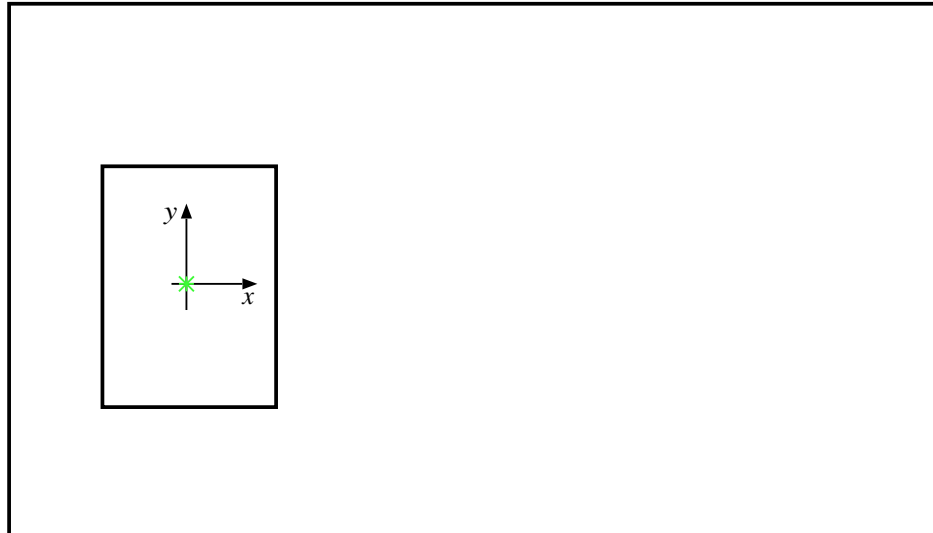


right (cupboard, lamp)

Select point randomly in room.

Generating Depictions. Case 1.

(1) (2) (3)

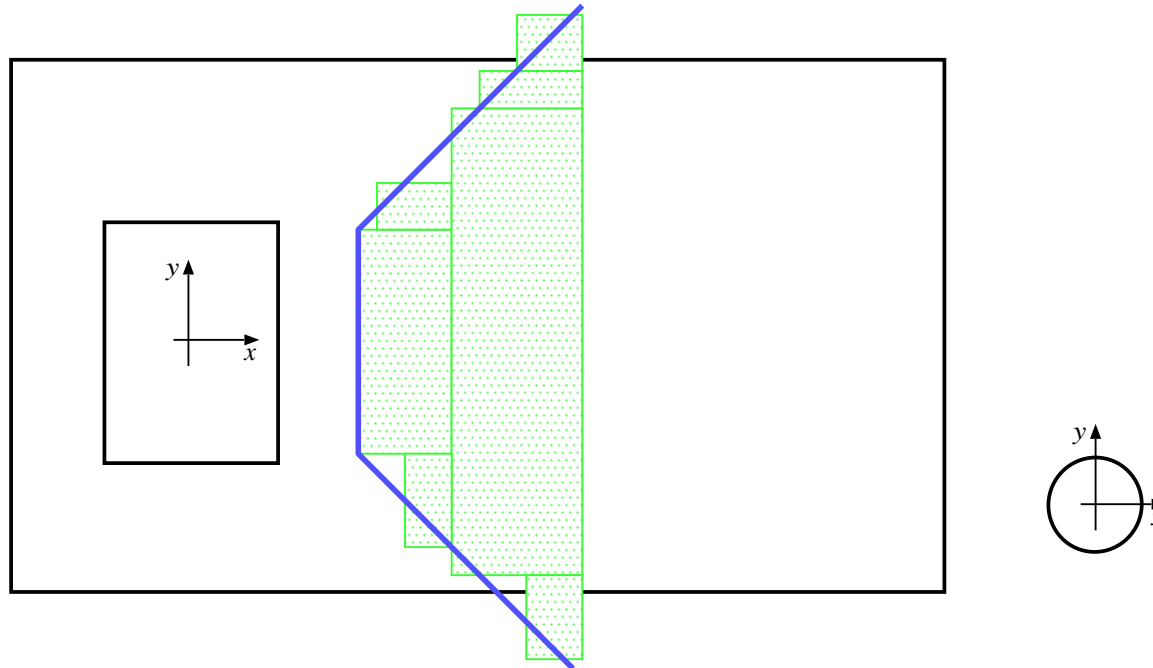


right (cupboard, lamp)

Place cupboard.

Generating Depictions. Case 1+2.

(1) (2) (3)

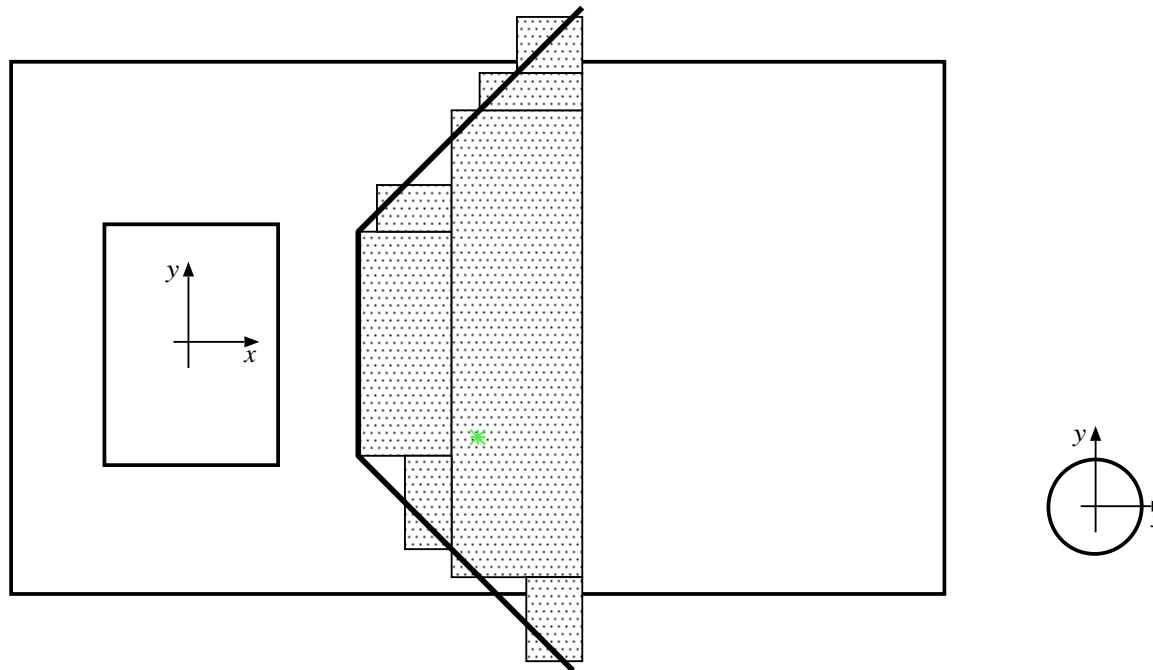


right (cupboard, lamp)

Use CAL5 decision tree.

Generating Depictions. Case 1+2.

(1) (2) (3)

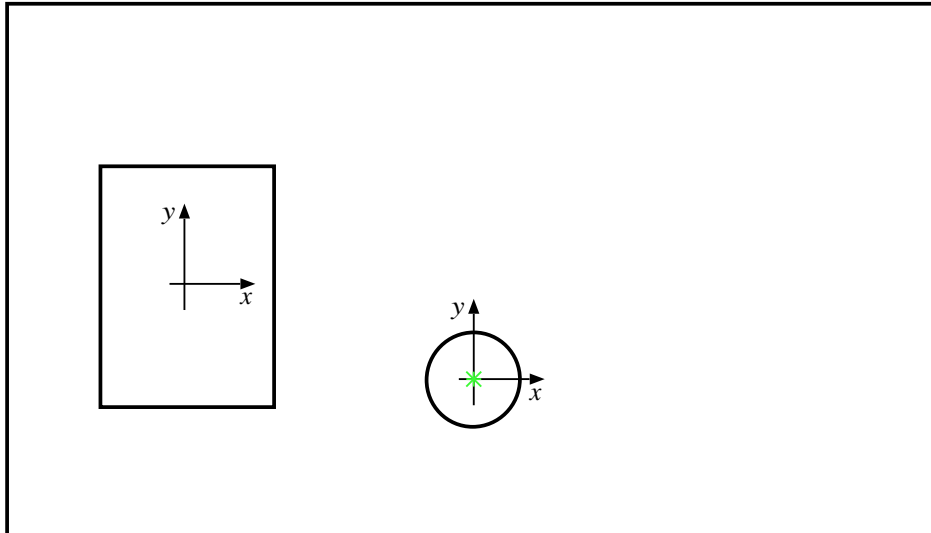


right (cupboard, lamp)

Select point in the class A region of the tree.

Generating Depictions. Case 1+2.

(1) (2) (3)

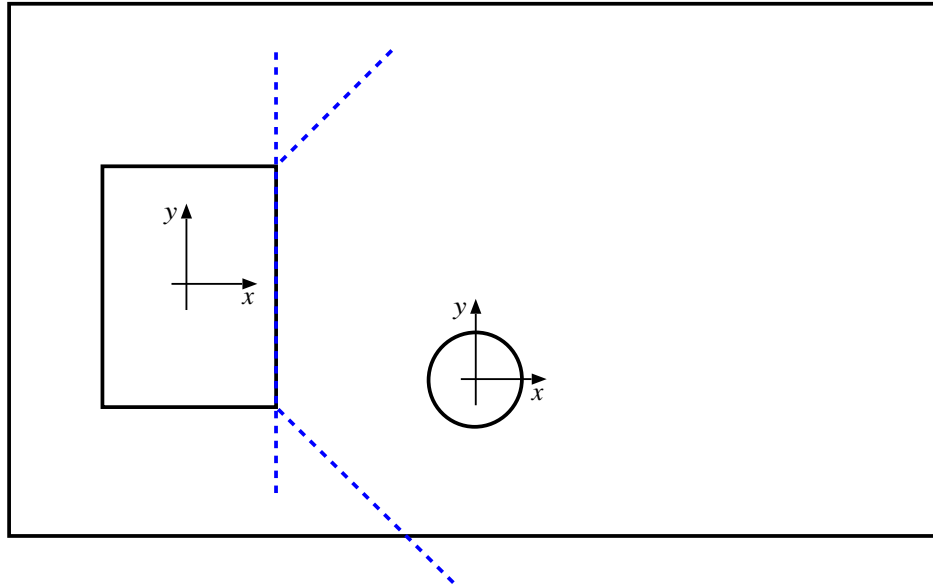


right (cupboard, lamp)

Place lamp.

Generating Depictions. Case 1+2.

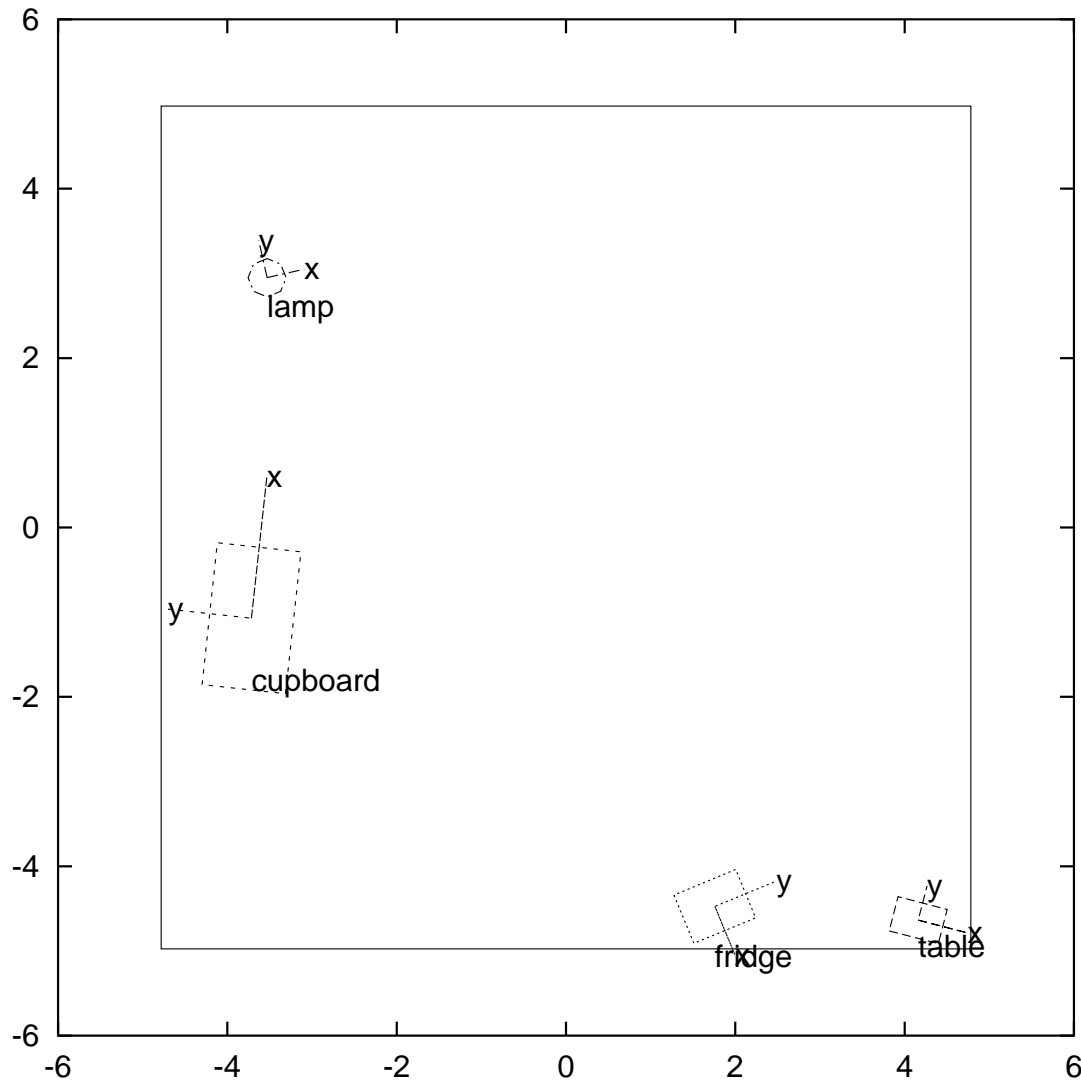
(1) (2) (3)



right (cupboard, lamp) holds.

Generating Depictions. Results.

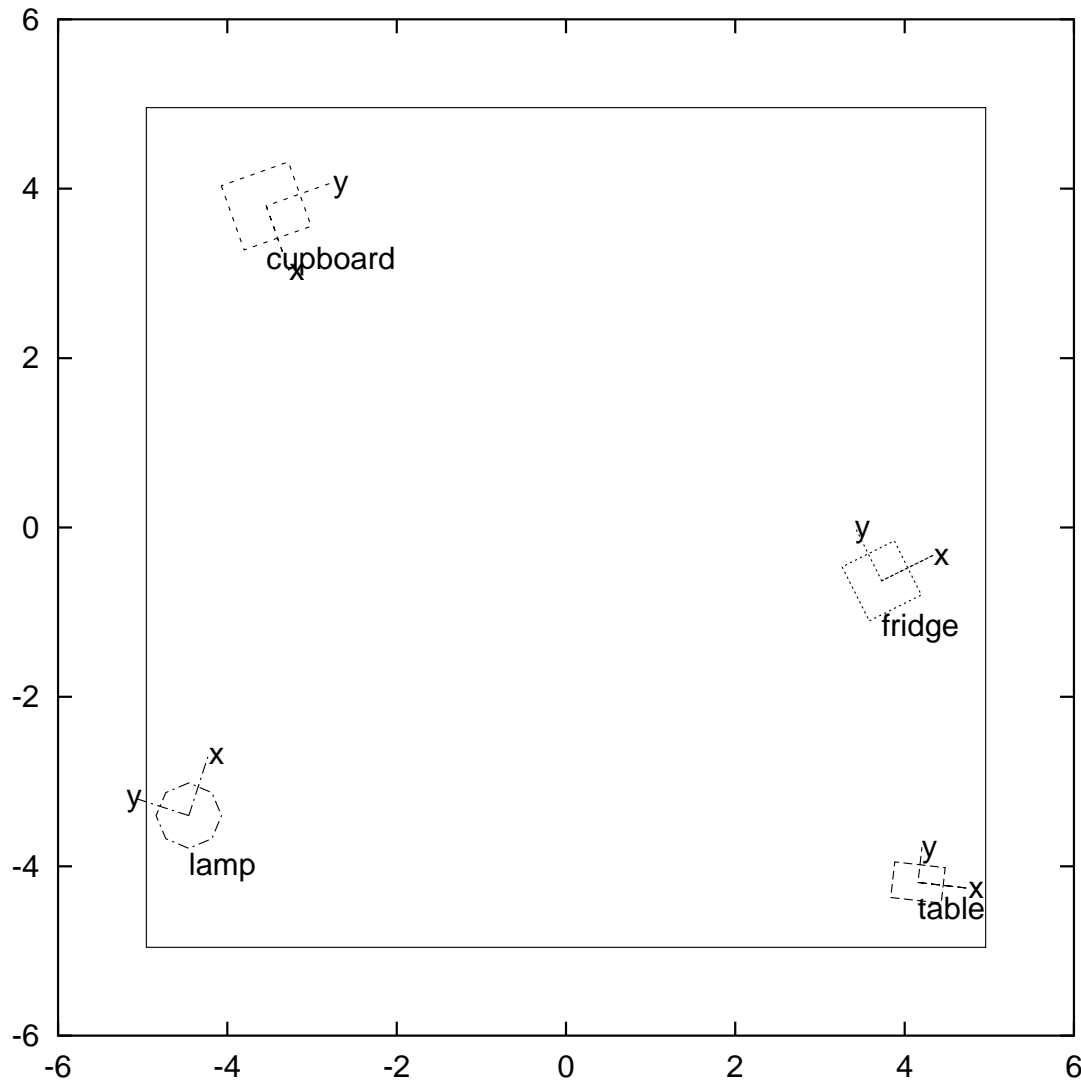
(1) (2) (3)



```
right (cupboard, lamp)
left (fridge, lamp)
left (table, lamp)
```

Generating Depictions. Results.

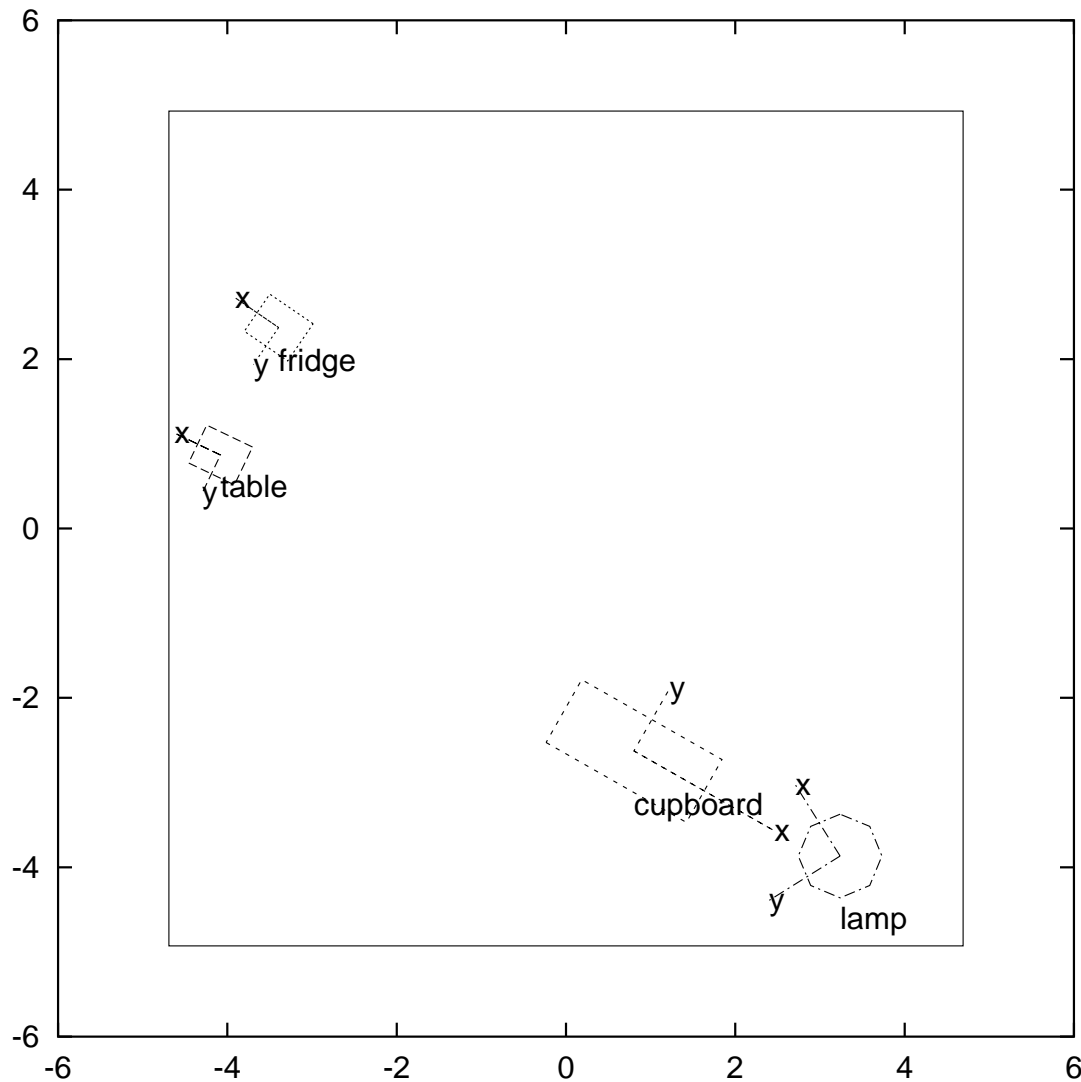
(1) (2) (3)



```
right (cupboard, lamp)  
left (fridge, lamp)  
left (table, lamp)
```

Generating Depictions. Results.

(1) (2) (3)



```
right ( cupboard , lamp )  
left ( fridge , lamp )  
left ( table , lamp )
```

- Many Internet applications are text based.
- Often spatial descriptions in emails or news.
- Aim: Generating appropriate depictions.
- Machine learning approach:
 - Quantitative description of spatial relations.
 - Learning decision boundaries of spatial relations.
 - Generating depictions using the decision trees.

Change of the basis:

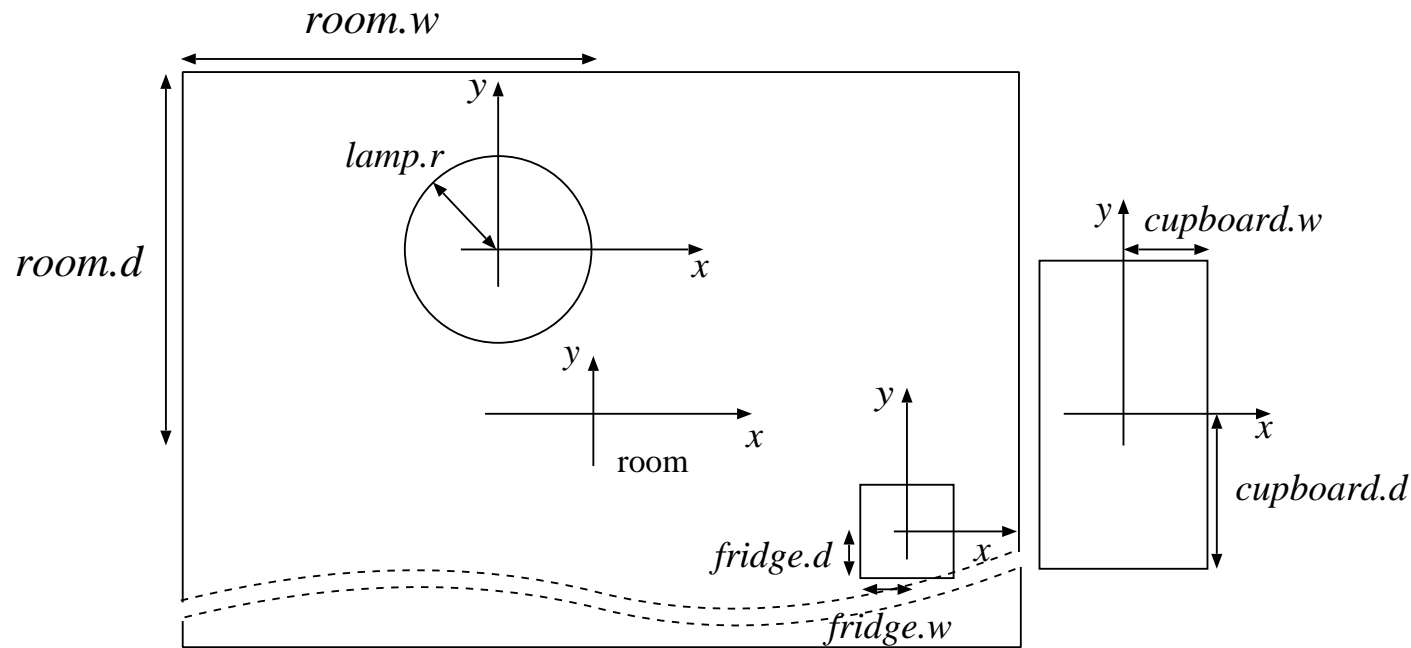
$$\mathbf{p}_k^i = \mathbf{P}_j^i \mathbf{p}_k^j$$

\mathbf{P}_j^i Homogeneous transformation matrix:

$$\mathbf{P}_j^i = \begin{pmatrix} \cos \theta_j^i & -\sin \theta_j^i & 0 & \Delta x_j^i \\ \sin \theta_j^i & \cos \theta_j^i & 0 & \Delta y_j^i \\ 0 & 0 & 1 & \Delta z_j^i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Misc. Machine Learning. Depiction Generation: 3.

(1) (2) (3)

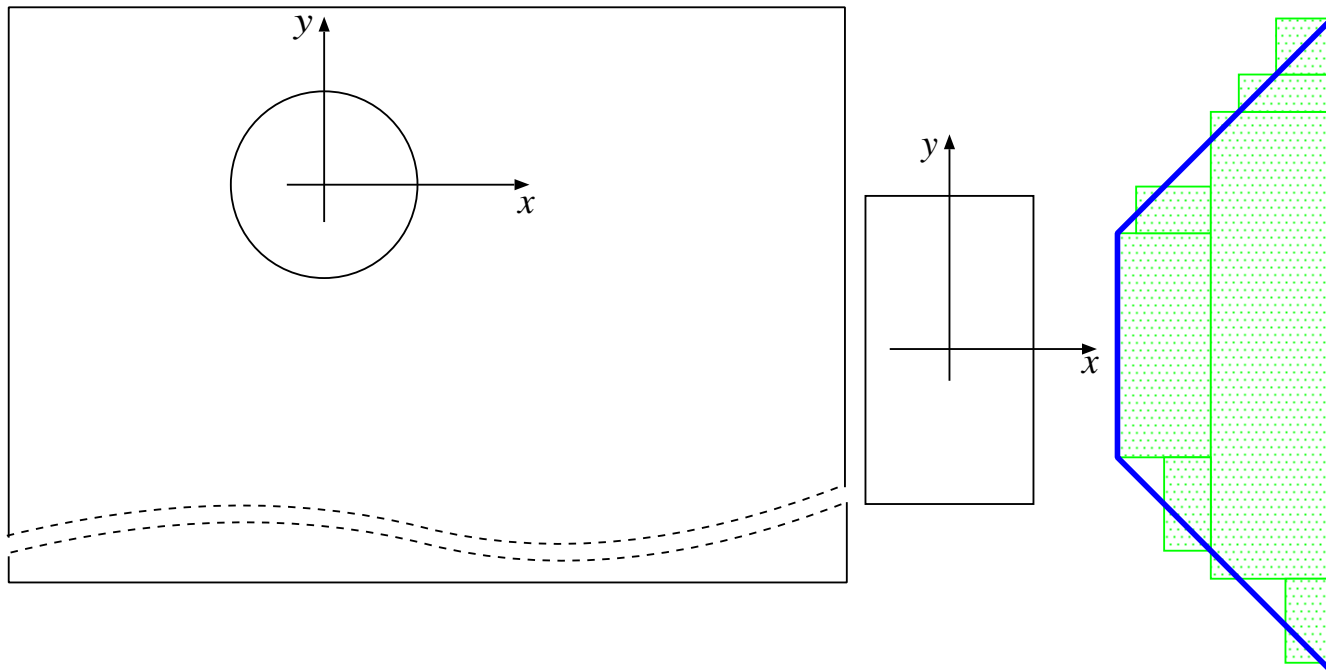


right (cupboard, lamp)

Lamp already placed.

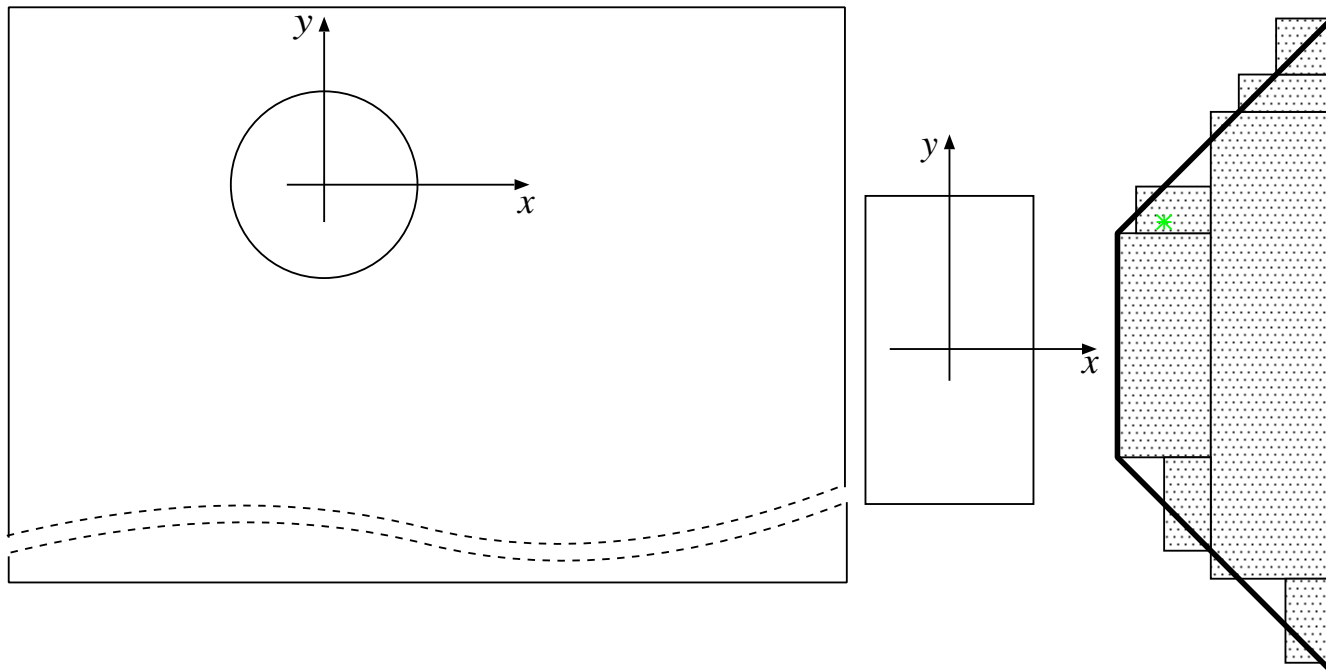
Misc. Machine Learning. Depiction Generation: 3.

(1) (2) (3)



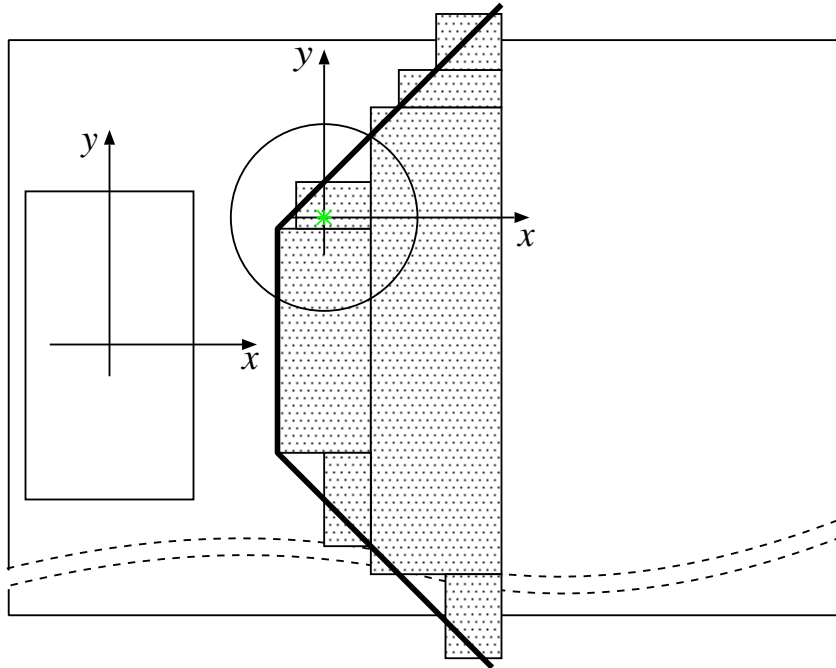
right (cupboard, lamp)

Use CAL5 decision tree.



right (cupboard, lamp)

Chose point in the class A region of the tree.



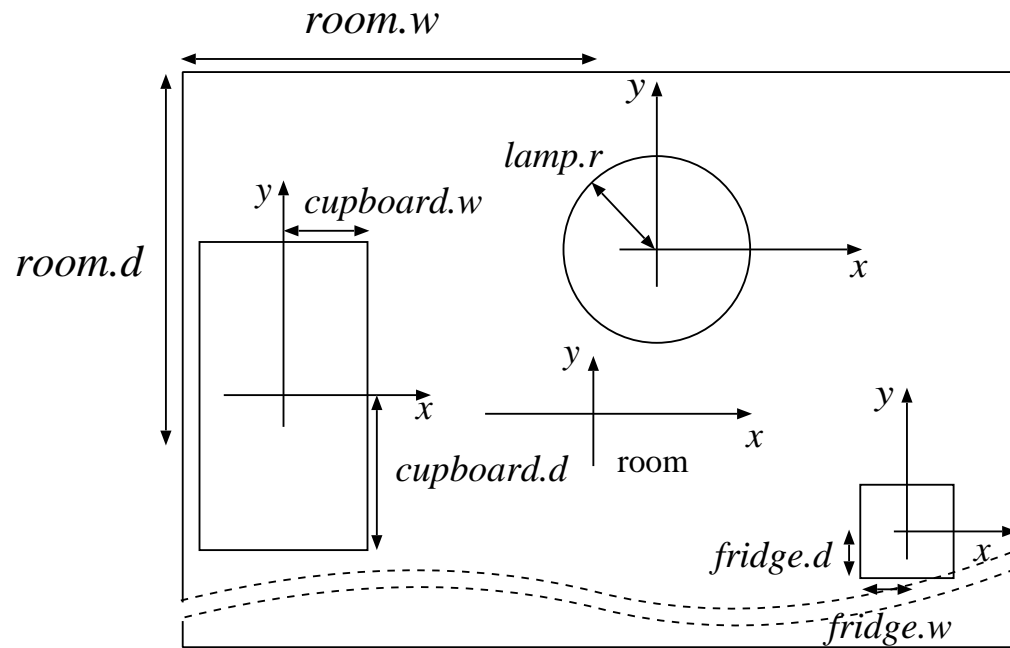
right (cupboard, lamp)

Map point to origin of lamp, backtransformation:

Cupboard fixed.

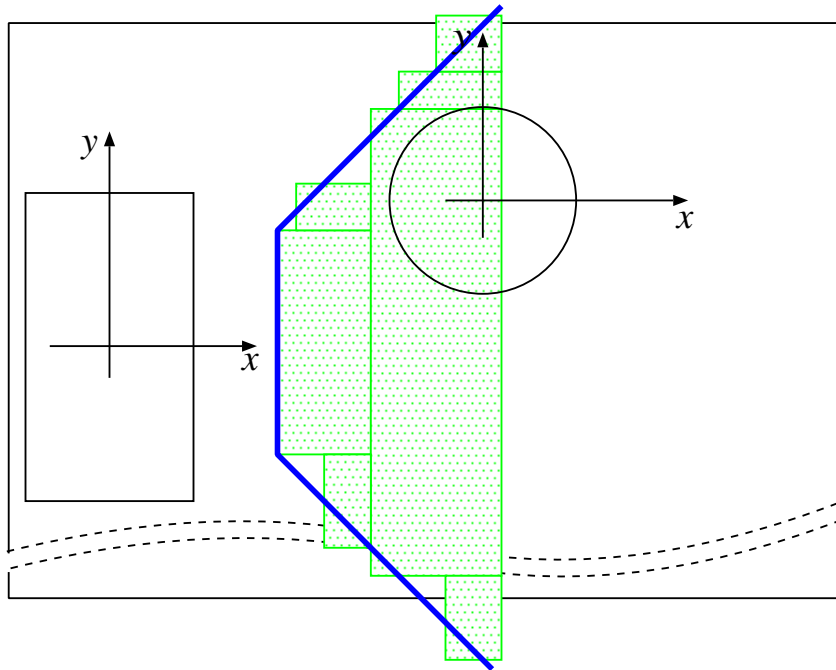
Misc. Machine Learning. Depiction Generation: 4.

(1) (2) (3)



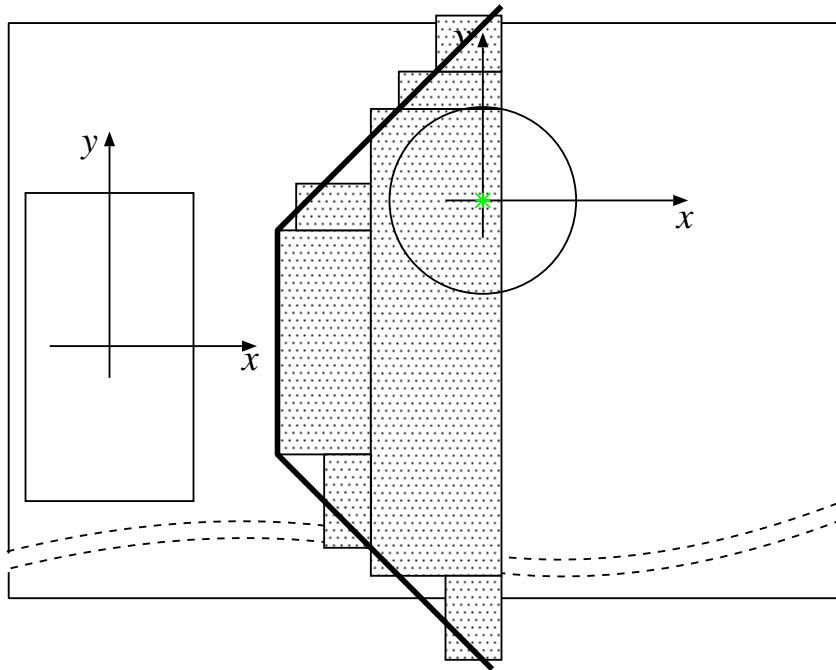
`right (cupboard, lamp)`

Both objects already placed.



right (cupboard, lamp)

Use CAL5 decision tree.



`right (cupboard, lamp)`

Origin of lamp is in the class A region of the tree:

`right (cupboard, lamp)` holds.

- 'Active Learning': Generating appropriate data sets.
- Using other machine learning algorithms.
- Investigating inference mechanisms.